Working together is encouraged, share ideas not calculations. Explain your steps. I will collect some subset of these problems. A page to write answers on will be distributed in class the day before the Mission is due.

Problem 144 Please read Sections 3.3 - 3.10 of the Lecture Notes.

Problem 145 Use the rules of differentiation to find the following derivatives without using the product or quotient rules.

(a.) $\frac{d}{dx}\left[\frac{1}{3\sqrt[5]{x^2}}\right]$

(b.) $\frac{d}{dx}\left[\frac{2}{7x^3}\right]$

(c.) $\frac{d}{dx}\left[4\pi^2\right]$

(d.) $\frac{d}{dx}\left[\sqrt[3]{5x^8}\right]$

(e.) $\frac{d}{dx}\left[\frac{5x^2-2x^{-4}}{\sqrt[3]{x^2}}\right]$

(f.) $\frac{d}{dx}\left[2x^3+7x\right]$

(g.) $\frac{d}{dx}\left[2\cos x-7\sin x\right]$

(h.) $\frac{d}{dx}\left[(3x+1)x^3\right]$

(i.) $\frac{d}{dx}\left[x^7+e^x\right]$

(j.) Calculate, given a,b,c constants: $\frac{d}{dx}\left[ax^2+bx+c\right]$

Problem 146 Suppose the position of a ninja is given by $x(t)=3t^2+2t+1$. Calculate the instantaneous velocity of the ninja at time $t=1$ by calculating $x'(1)$.

Problem 147 Use the product and quotient rules to calculate the following derivatives (do not simplify, assume variables not being differentiated are held constant in the differentiation)

(a.) $\frac{d}{dx}\left[(2x^3+6x-5)(4x^2+7)\right]$

(b.) $\frac{d}{dx}\left[\frac{3ax^2-2}{a^3+2x}\right]$

(c.) $\frac{d}{da}\left[\frac{3ax^2-2}{a^3+2x}\right]$

Problem 148 Use the chain rule to calculate the following derivatives (do not simplify, assume variables not being differentiated are held constant in the differentiation)

(a.) $\frac{d}{dx}\left[\frac{2}{x^2+5x}\right]$

(b.) $\frac{d}{d\beta}\left[\sin^2(3\beta)\right]$

(c.) $\frac{d}{dx}\left[\frac{7}{\overline{5\sqrt[3]{x}+\sin x}}\right]$

(d.) $\frac{d}{dx}\left[\sqrt{\tan(3x^2+2)}\right]$

(e.) $\frac{d}{dw}\left[\sqrt{w^3+\sqrt{w^2+\sqrt{w}}}\right]$

Problem 149 Differentiate by using the rules of differential calculus. If possible, simplify answers by factoring out any common term.

(a.) $\frac{d}{dx}\left[x^2e^{-2x+3}\right]$

(b.) $\dfrac{d}{d\beta}\left[\ x^3\tan(2x^2)\ \right]$

(c.) $\dfrac{d}{dx}\left[\ x\cos(x^2)\sin(x^3)\ \right]$

(d.) $\dfrac{d}{dx}\left[\ \sqrt{x}\cosh(\sqrt{x})\ \right]$

(e.) $\dfrac{d}{dx}\left[\ (x+3)^{10}3^{10x}\ \right]$

(f.) $\dfrac{d}{dx}\left[\ \dfrac{x\sin x}{\cos^3(x^2)}\ \right]$

(g.) $\dfrac{d}{dx}\left[\ e^{-x}\sin(x+\sqrt{x})\ \right]$

(h.) $\dfrac{d}{dx}\left[\ \left(\dfrac{2x+3}{2x^2+5x+1}\right)e^{3x}\ \right]$

(i.) $\dfrac{d}{dx}\left[\dfrac{\cos^2 x+\sin^2 x}{\tan^2 x+1}\right]$

(j.) $\dfrac{d}{dx}\left[\dfrac{e^x+e^{-x}}{e^{3x}}\right]$

Problem 150 Calculate

$$\frac{d}{dx}(x^2\cos x)$$

Problem 151 Calculate

$$\frac{d}{dx}(xe^x\cos x)$$

Problem 152 Calculate

$$\frac{d}{dx}(x^7+e^x\sin x)$$

Problem 153 Calculate:

$$\frac{d}{dx}\left[x^3+x^2+2\right]^3=$$

Problem 154 If $f(x)=x\sqrt{x}$ calculate $f'(x)$ and $f''(x)$.

Problem 155 Calculate

$$\frac{d}{dx}\left[\frac{2x+3}{x^2+x^4}\right]$$

Problem 156 Calculate

$$\frac{d}{dx}\left[\frac{\sin x}{x+e^x}\right]$$

Problem 157 Calculate

$$\frac{d}{dx}\left[\frac{1}{\sqrt{x^2+1}}\right]$$

Problem 158 Suppose f is a differentiable function on $\mathbb{R}$ and let $g(x)=x^3+e^{-2x}$. Calculate the following (leave your answer in terms of $f'(x)$ and f as appropriate)

$$\frac{d}{dx}g(f(x))$$

Problem 159 Calculate

$$\frac{d}{dx}\left[e^x \sin(3+x^2)\right]$$

Problem 160 Calculate

$$\frac{d}{dx}\sin\left(3+x2^x\right)$$

Problem 161 We can use notation $e^x = exp(x)$ where helpful. Calculate:

$$\frac{d}{dx}exp\left(\sqrt{\cos x}\right)$$

Problem 162 Calculate:

$$\frac{d}{dx}\left[\frac{x^3}{6-\sqrt{2x+7}}\right] =$$

Problem 163 Let a, b be differentiable functions. Calculate:

$$\frac{d}{dx}\sin\left[ab\right]$$

Problem 164 Assume $a, b, c \in \mathbb{R}$ are constants. Calculate:

(a.) $\frac{d}{dx}\left[x^a + \sqrt{bx^2+c}\right]$

(b.) $\frac{d}{d\theta}\tan\big(a\theta^2 + b\theta + c)\big)$

Problem 165 Let $f(x) = x^4 + 3x^3 + 2x + 1$. Calculate $f^{(n)}(x)$ for $n = 1, 2, 3, \dots$.

Problem 166 Let $f(x) = \sin(2x)$. Calculate $f^{(n)}(x)$ for $n = 1, 2, 3, \dots$. It is helpful to break-up your answer into cases.

Problem 167 Differentiation is a great way to derive new identities from old. Suppose you are given that $\sin(x+a) = \sin x \cos a + \cos x \sin a$. Differentiate this identity with respect to x holding a constant to find an identity for $\cos(x+a)$.

Problem 168 Suppose that $x = t^3 - 3t$ is the equation of motion for a particle travelling in one dimension with position x at time t. Find:

(a.) velocity at time t

(b.) acceleration at time t

(c.) acceleration when the velocity is zero (there may be one, none or several answers)

Problem 169 Verify the following rules of differentiation:

(a.) $\frac{d}{d\theta}\tan\theta = \sec^2\theta$

(b.) $\frac{d}{d\theta}\sec\theta = \sec\theta\tan\theta$

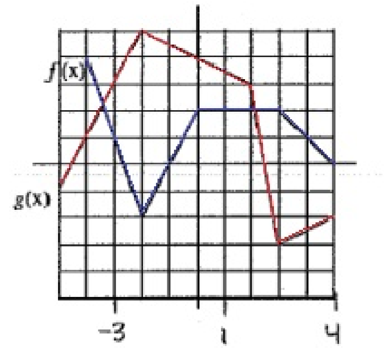
(c.) $\frac{d}{d\theta}\cot\theta = -\csc^2\theta$

(d.) $\frac{d}{d\theta}\csc\theta = -\csc\theta\cot\theta$

Problem 170 Calculate $\frac{d}{d\phi}\tanh\phi$ and $\frac{d}{d\phi}\text{sech}\,\phi$.

Problem 171 Suppose $f(x) = \sqrt{x}g(x)$ where $g(4) = 5$ and $g'(4) = -3$, find $f'(4)$.

Problem 172 Given the graph of f and g pictured below find:



(a.) $(fg)'(-3)$

(b.) $\left(\frac{g}{f}\right)'(1)$

(c.) $\left(\frac{f}{g}\right)'(4)$

Problem 173 Let $h(x) = g(f(x))$ and $k = f(g(x))$. Once more use the graph in the previous problem to calculate:

(a.) $(h)'(-3)$

(b.) $k'(-3)$

(c.) $h'(-1)$

Problem 174 Consider the casewise-defined function $f(x) = |4x + 6\sqrt[3]{x}|$.

(a.) calculate $\frac{df}{dx}$ and write answer as an explicit casewise-defined function

(b.) find the equation of the tangent line to $y = f(x)$ at $x = -1$.

Problem 175 Find the equation of the tangent line to $y = 2^{\frac{4}{x}}$ at $x = 2$.

Problem 176 A standard method to calculate the derivative of an inverse function is as follows:

(i.) write $y = f^{-1}(x)$

(ii.) act on the equation by f to obtain $f(y) = x$

(iii.) differentiate the equation $f(y) = x$ to obtain $f'(y)\frac{dy}{dx} = 1$

(iv.) solve for $\frac{dy}{dx}$ and put everything in terms of x by using known identities for the given function and you're done.

Use the above method to derive the formulas for the following derivatives:

(a.) $\dfrac{d}{dx}\tan^{-1} x$

(b.) $\dfrac{d}{dx}\cos^{-1} x$

(c.) $\dfrac{d}{dx}\sinh^{-1} x$

(d.) $\dfrac{d}{dx}\ln x$

(e.) given $a > 1$, $\dfrac{d}{dx}\log_a x$

(f.) $\dfrac{d}{dx}\sec^{-1} x$

(g.) $\dfrac{d}{dx}\tanh^{-1} x$

(h.) $\dfrac{d}{dx}\text{sech}^{-1} x$

Problem 177 Differentiate the given function. Simplify if possible. Use implicit technique if necessary.

(a.) $y = \ln x \sin x$

(b.) $y = \ln(\sin x)$

(c.) $y = \ln|2x^3 + 5x - 3|$

(d.) $f(x) = \dfrac{\ln x}{x^3}$

(e.) $g(x) = \sqrt{\cos[5x + \ln(x^2 + 3)]}$

(f.) $y = 2^x \sin x$

(g.) $y = 2^{\sin x}$

(h.) $y = (2^x + x^2)^5$

(i.) $f(x) = x^{\sin x}$

(j.) $g(x) = x^{x^2}$

(k.) $y = \cosh(\ln x^2)$

Problem 178 Show the function $f(x) = 3x + 4\sqrt{x}$ has no tangent line with slope 1.

Problem 179 Find any points on the curve $y = 3x + 4\sqrt{x}$ where the slope of the curve is 7.

Problem 180 Find an equation of the normal line to the curve $y = \frac{2}{x} + x$ at $x = 2$. At which point does the normal line intersect the curve again ?

Problem 181 Find all values of x where the graph of $y = (2x - 7)^2(3x + 2)^8$ has horizontal tangent lines.

Problem 182 Find the equation of the tangent line to $y = x^2 e^x$ at $x = 1$.

Problem 183 In the interval $[0, 2\pi]$, where (what values of x) does $y = \frac{1}{\sin x + \cos x}$ have horizontal tangent lines ?

Problem 184 In the interval $[0, \pi]$, what values of x does $f(x) = \frac{4}{3}\sin^3 x - \cos x$ have horizontal tangent lines ?

Problem 185 What is the slope of $y = f(x) = ax^2 + bx + c$ at the point $(x_o, f(x_o))$? What is the significance of the point where $f'(x_o) = 0$?

Problem 186 Let $f(x) = x^4$. Find the Caratheodory function $\phi(x)$ for which

$$f(x) - f(a) = \phi(x)(x - a)$$

and derive $f'(a) = 4a^3$ from the theorem of Caratheodory.

Problem 187 Suppose f is differentiable at a where $a \in int(dom(f))$. Prove f is continuous at $x = a$.

Problem 188 Suppose $f'(a)$ and $g'(a)$ both exist. Derive the differentiation rules

$$(f + g)'(a) = f'(a) + g'(a) \qquad \& \qquad (fg)'(a) = f'(a)g(a) + f(a)g'(a).$$

Problem 189 Suppose $f_1'(a), f_2'(a) \dots, f_n'(a)$ exist. Derive the product rule for differentiating the product of n-fold functions; find the formula for

$$(f_1 f_2 \cdots f_n)'(a)$$

Problem 190 Assume you are given that $f'(a), g'(a)$ exists with $g'(a) \neq 0$ and f/g is differentiable at a. Let $h = f/g$ hence $f = hg$ and apply the product rule to h in order to derive the quotient rule[1].

Problem 191 Assume $f^{(n)}(a), g^{(n)}(a)$ exist for $n = 0, 1, 2, \dots$. We denote $f^{(0)}(a) = f(a)$ and $f^{(1)}(a) = f'(a)$. Furthermore, $f^{(n+1)}(a) = (f^{(n)})'(a)$ for all $n \in \mathbb{N}$. Derive the formula for $(fg)^{(n)}(a)$. It will be helpful to review the Binomial Theorem since the algebra here is very similar.

Problem 192 Let $f(x) = \sin x$ and $g(x) = \cos x$. Prove $f'(0) = 1$ and $g'(0) = 0$. See the lecture notes for the needed inequalities which supports a squeeze theorem argument to derive the desired limit of $\sin h/h$ as $h \to 0$.

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[1] Of course, it logically remains to show f/g is indeed differentiable with derivative given by the quotient rule, but that is already done in the lecture note. Thank me in that I didn't ask you to prove that, deriving the form of the rule is way easier than verification it works

Problem 193 Prove $f'(a) = \cos a$ and $g'(a) = -\sin a$ for all $a \in \mathbb{R}$ where $f(x) - \sin x$ and $g(x) = \cos x$. Your proof should procede from the limit definition of the derivative. You should assume the basic indeterminant limits about sine and cosine at zero are known from the previous problem. That is, you are given

$$\lim_{h \to 0} \frac{\sin h}{h} = 1 \qquad \& \qquad \lim_{h \to 0} \frac{\cos h - 1}{h} = 0.$$

Problem 194 One possible definition for e is that it is the number for which $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$. Prove $\frac{d}{dx} e^x = e^x$.

Problem 195 Let $a > 0$ then prove $\frac{d}{dx} a^x = (\ln a) a^x$ in two ways:

(a.) by writing $a^x = e^{kx}$ for appropriate choice of k and the chain-rule,

(b.) via implicit differentiation of an appropriate equation.