Working together is encouraged, share ideas not calculations. Explain your steps. I will collect some subset of these problems. A page to write answers on will be distributed in class the day before the Mission is due.

Problem 267 Please read Sections 5.3 - 5.6 of the Lecture Notes.

Problem 268 Suppose a box holds a volume of $32,000\ cm^3$. The base of the box is square side length x and the sides have height y. The top of the box is open. What values should we choose for x and y as to minimize the amount of material needed to construct the box ?

Problem 269 What is the maximum area of a rectangle inscribed between the parabola $y = 8 - x^2$ and the x-axis ? Use calculus to support your claim.

Problem 270 Consider a 10 meter length of wire which is cut into two pieces such that a length x piece is bent into a square and the remaining part is bent into a circle. Find maximum and minimum value for the net area bounded by both the square and the circle.[1]

Problem 271 Find points on the ellipse $4x^2 + y^2 = 4$ which are furthest from $(1, 0)$.

Problem 272 Find dimensions of the rectangle of largest area which may be inscribed in a circle of radius R.

Problem 273 A beehive is made of cells which are regular hexagonal prisms with a trihedral endcap with characteristic angle θ. It turns out the surface area of a cell in the beehive is given by:

$$S = 6sh - \frac{3}{2}s^2 \cot\theta + \left(\frac{3s^2\sqrt{3}}{2}\right)\csc\theta$$

where s, b, h are constants and $0 < \theta < \pi/2$. Find the angle θ which minimizes the S. This is actually observed in nature, however, there is technically a slight improvement possible in terms of minimizing the surface area in terms of pure geometry. You can read more at: https://en.wikipedia.org/wiki/Honeycomb if you're interested.

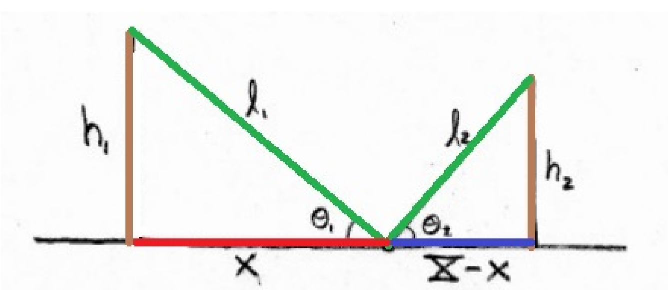
Problem 274 Solve the problems below with calculus:

(a.) Show a square has the greatest area of all the rectangles formed by a given perimeter P.

(b.) Show a square has the smallest perimeter of all the rectangles formed by a given area A.

Problem 275 Find the point on the line $y = 4x + 7$ which is closest to the origin.

Problem 276 Two poles of unequal height $h_1 \neq h_2$ are secured by a very taught rope to a point between the poles as pictured below. Show that the shortest length of rope needed to secure the poles occurs when $\theta_1 = \theta_2$. Let X be the distance betweent the base of the poles.[2]



Problem 277 Find two positive real numbers x, y whose product is 100 and the sum of the second number and four times the first number is a minimum.

Problem 278 Suppose the speed of a race car is given by $v(t) = 200t - 4t^2$ for $0 \leq t \leq 50$ where t is in seconds and v is in m/s. Find the maximum speed of the car.

[1] hint: $0 \leq x \leq 10$ is the physically reasonable range for x given the statement of the problem.

[2] when I worked out this problem I used x as a variable to set-up the calculus, there may be a wiser path.

Problem 279 Let us review some limits at $\pm\infty$ which can be solved without L'hopital's Rule:

(a.) $\displaystyle\lim_{x\to-\infty} \frac{2x^3-3x^2+5}{3x^3-2x^2-5}$

(b.) $\displaystyle\lim_{x\to-\infty} \frac{\sqrt{5x^6-2x^3+3}}{9x^3+8}$

(c.) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{7x^2+x-2}}{2x-3}$

(d.) $\displaystyle\lim_{x\to\infty} \frac{\sqrt{9x^4-1}}{2x^2-x-10}$

(e.) $\displaystyle\lim_{x\to\infty} \left[2x-\sqrt{4x^2+3x-1}\right]$

(f.) $\displaystyle\lim_{x\to-\infty} \left[2x^3+5x^2\right]$

Problem 280 Find the horizontal asymptotes for the graphs given below:

(a.) $\displaystyle y=\frac{3x^2+x-9}{2x^2-x-15}$

(b.) $\displaystyle y=\frac{\sqrt{2x^2-7}}{x-8}$ (be careful)

Problem 281 Use L'hopital's Rule to calculate the following limits:

(a.) $\displaystyle\lim_{x\to0} \left[x^2e^{\frac{1}{x^2}}+\tan^{-1}(\csc^2(x))\right]$

(b.) $\displaystyle\lim_{x\to0} \left[2^{px-qx}\right]$ where p, q are constants.

(c.) $\displaystyle\lim_{x\to\infty} \frac{1}{\left[1+\frac{1}{x}\right]^{bx}}$ where b is a constant.

Problem 282 Calculate the limits via L'Hopital's Rule and appropriate algebra.

(a.) $\displaystyle\lim_{x\to1} \left[\frac{x^2-1}{x^2-x}\right]$

(b.) $\displaystyle\lim_{t\to0} \left[\frac{e^t-1}{t^3}\right]$

(c.) assume $q\neq0$, $\displaystyle\lim_{x\to0} \frac{\tan(px)}{\tan(qx)}$

(d.) $\displaystyle\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$

(e.) $\displaystyle\lim_{x\to\infty} \frac{e^x}{x^3}$

(f.) $\displaystyle\lim_{x\to0} \frac{e^x-1-x}{x^2}$

(g.) $\displaystyle\lim_{t\to0} \frac{5^t-3^t}{t}$

(h.) $\displaystyle\lim_{x\to0} \frac{\sin^{-1}(x)}{x}$

(i.) $\displaystyle\lim_{t\to\infty} x^3e^{-x^2}$

(j.) $\displaystyle\lim_{t\to\infty} \left(\sqrt{x^2+x}-x\right)$

Problem 283 Calculate the limits via L'Hopital's Rule and appropriate algebra.

(a.) for $n \in \mathbb{N}$, $\lim_{x\to\infty} \frac{e^x}{x^n}$

(b.) for $p > 0$, $\lim_{x\to\infty} \frac{\ln x}{x^p}$

(c.) for $a, b \neq 0$, $\lim_{x\to 1} \frac{x^a - 1}{x^b - 1}$

(d.) $\lim_{x\to 0} \frac{x + \tan x}{\sin x}$

(e.) $\lim_{x\to \pi} \frac{1}{x \cot x}$

(f.) $\lim_{x\to\infty} \frac{\ln(ln(x))}{x}$

(g.) for m, n constants, $\lim_{x\to 0} \left(\frac{\cos(mx) - \cos(nx)}{x^2}\right)$

(h.) $\lim_{x\to 0} \frac{x}{\tan^{-1}(4x)}$

(i.) $\lim_{x\to \frac{\pi}{2}^-} \sec(7x)\cos(3x)$

(j.) $\lim_{x\to 1^+} \left[(x-1)\tan\left(\frac{\pi x}{2}\right)\right]$

(k.) $\lim_{x\to 0} (\csc(x) - \cot(x))$

(l.) $\lim_{x\to 1} \left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$

Problem 284 Calculate the limits of the following indeterminant powers via L'Hopital's Rule and appropriate algebra.

(a.) $\lim_{x\to 0} (1-2x)^{1/x}$

(b.) $\lim_{x\to\infty} \left(x^{1/x}\right)$

(c.) $\lim_{x\to 0^+} (\cos x)^{1/x^2}$

(d.) $\lim_{x\to 0^+} (\sin x)^{\tan x}$

(e.) $\lim_{x\to\infty} x^{\frac{\ln 2}{1+\ln x}}$

Problem 285 Calculate $\lim_{x\to\infty} \frac{210x^2 - 6x + 4}{5x^2 + 15x - 8}$.

Problem 286 Calculate $\lim_{x\to\infty} \frac{x^2 + 3x - 8}{e^x}$

Problem 287 Calculate $\lim_{x\to 0} \frac{x^2}{\ln(3x)}$

Problem 288 Calculate $\lim_{x\to 0} \frac{\ln(6x^3 + 2)}{\ln(x^3 + 3x + 8)}$

Problem 289 Calculate $\lim_{x\to 2} \left[\frac{1}{\ln(x-1)} - \frac{1}{x-2}\right]$

Problem 290 Calculate $\lim_{x\to\infty} e^x \sin\left(e^{-x}\right)$

Problem 291 Let a, b be positive constants. Calculate $\lim_{x\to 0} \frac{\sin(ax)}{\sin(bx)}$.

Problem 292 Show $e^t = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{xt}$.

Problem 293 Given $\tan^{-1}(x) < f(x) < \frac{\pi}{2}\tanh(x)$ for $x > 42$ calculate $\lim_{x\to\infty} f(x)$.

Problem 294 Find the Taylor poynomial of order 4 centered at the given a for the functions below:

(a.) $f(x) = x^4 + 4$, with center $a = 2$,

(b.) $g(x) = \tan x$, with center $a = 0$,

(c.) $h(x) = \dfrac{1}{1-x}$, with center $a = 0$.

Problem 295 We say a polynomial $p(x)$ has a zero x_o with multiplicity m if $p(x) = (x - x_o)^m g(x)$ where $g(x_o) \neq 0$ is a polynomial. Use values of derivatives of $p(x)$ at x_o to give a condition on the derivatives of $p(x)$ which make x_o a zero with multiplicity m.

Problem 296 Consider $f(x) = x^5 + x^4 - 6x^3 + 2x^2 + 5x - 3$. Use the result of the previous problem to show $f(x)$ has the zero $x_o = 1$ with multiplicity 3. Finally, completely factor $f(x)$ over $\mathbb{R}$.