

Working together is encouraged, share ideas not calculations. Explain your steps. I will collect some subset of these problems. A page to write answers on will be distributed in class the day before the Mission is due.

Problem 323 Please read Sections 6.5 and 6.6 of the Lecture Notes.

Problem 324 Calculate $\int x^3 \sin(x^4 + 3) dx$

Problem 325 Calculate $\int \frac{\cos x dx}{1 + \sin^2 x}$

Problem 326 Calculate $\int \frac{\cosh \sqrt{x}}{2\sqrt{x}} dx$

Problem 327 Calculate $\int \tan^2(3x) dx$

Problem 328 Calculate $\int e^x \sqrt{3 + e^x} dx$

Problem 329 Calculate $\int \frac{(1 + 2^x)dx}{3 + x + 2^x}$

Problem 330 Calculate $\int \frac{x^2 dx}{x + 1}$

Problem 331 Calculate $\int \frac{x dx}{x^2 + 6x + 13}$

Problem 332 Calculate $\int_0^{\pi/7} \sin^3(7x) dx$

Problem 333 Calculate $\int_{8/3}^3 (3x - 8)^{16} dx$

Problem 334 Calculate $\int_0^1 x^2(1 + x^3)^{10} dx$

Problem 335 Calculate $\int_{\ln 3}^{\ln 4} \frac{e^x dx}{\sqrt{1 - e^{2x}}}$

Problem 336 Calculate $\frac{d}{dt} \int_3^t \sqrt{\cos \tau + \tau^2} d\tau$.

Problem 337 Calculate $\frac{d}{dx} \int_{x^2}^{e^x} \ln(t^2 + 3) dt$.

Problem 338 Let $a > 0$. Suppose f is an even function with $\int_0^a f(x) dx = 7$. Also, let g be an odd function. Calculate:

$$\int_{-a}^a [3f(x) + g(x)] dx.$$

Problem 339 Calculate, by means of an appropriate substitution, the integrals below:

(a.) $\int 2x(1 + x^2)^{41} dx,$

(b.) $\int \cos x \cosh(\sin x) dx,$

- (c.) $\int \frac{x^7 + x^3}{1 + x^8} dx,$
- (d.) $\int x \sin(x^2 + 3) dx,$
- (e.) $\int \tan(4x + 7) dx,$
- (f.) $\int \frac{2x dx}{1 + x^4},$
- (g.) $\int \frac{dx}{e^{-x} + e^x},$

Problem 340 Calculate:

- (a.) $\int_0^1 (3x + 17)^{41} dx,$
- (b.) $\int_{\ln \pi - \ln 2}^{\ln \pi} e^x \cos(e^x) dx,$
- (c.) $\int_0^2 (2x + 1) \sin(x^2 + x) dx,$
- (d.) $\int_0^1 x \sqrt{x + 3} dx,$
- (e.) $\int_3^4 \frac{x}{x + 2} dx,$
- (f.) $\int_1^2 \frac{dx}{2x - 7},$
- (g.) $\int_6^{25} \frac{dx}{\sqrt[3]{x + 2}},$
- (h.) $\int_{\sqrt{\ln(3)}}^{\sqrt{\ln(7)}} x \exp(x^2) dx,$

Problem 341 Calculate:

- (a.) $\int \cos^3(4x) dx,$
- (b.) $\int_0^\pi \sin^3(\theta) d\theta,$
- (c.) $\int \sin^2(3x - 1) dx,$
- (d.) $\int_0^\pi \sin^2(\theta) d\theta,$
- (e.) $\int \sin(t) \cos^4(t) dt,$
- (f.) $\int \sec^3 x \tan x dx,$
- (g.) $\int \sin(3x) \cos(2x) dx.$
- (h.) $\int \frac{\cosh y}{\sinh^2 y} dy$
- (i.) $\int \cosh^3 x dx$

Problem 342 Calculate the integrals below by making an appropriate substitution: ¹

(a.) $\int x(4 + x^2)^{10} dx$

(b.) $\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx$

(c.) $\int e^{\sin \theta} \cos \theta d\theta$

(d.) $\int x e^{-x^2+8} dx$

(e.) $\int (2 - x)^6 dx$

(f.) $\int \frac{x dx}{x^2 + 1}$

(g.) $\int t^2 \cos(1 - t^3) dt$

(h.) $\int \sqrt[3]{3 - 5y} dy$

(i.) $\int \frac{\tan^{-1}(x) dx}{1 + x^2}$

(j.) $\int \frac{1}{x^2} \cos(\pi/x) dx$

(k.) $\int \frac{\sin x dx}{1 + \cos^2 x}$

(l.) $\int \sqrt{\cot x} \csc^2 x dx$

(m.) $\int \frac{1 + x}{1 + x^2} dx$

Problem 343 Let $u = \sec \theta + \tan \theta$ in order to calculate $\int \sec \theta d\theta$.

Problem 344 Calculate $\int \csc \theta d\theta$.

Problem 345 Integrate $\int 2^x \cos^3(2^x) \sin(2^x) dx$.

Problem 346 Integrate $\int \frac{2^x dx}{4^x + 1}$.

Problem 347 Integrate $\int x^2 \cosh^3(x^3 + 1) \sinh(x^3 + 1) dx$.

Problem 348 Integrate $\int \frac{dx}{e^x \sqrt{1 - e^{-2x}}}$.

Problem 349 Integrate $\int \frac{\sin x dx}{1 + \cos^2 x}$.

Problem 350 Integrate $\int \frac{\sin x dx}{1 - \cos^2 x}$.

Problem 351 Integrate $\int \frac{x^5 dx}{1 - x^{12}}$.

Problem 352 Integrate $\int \frac{x^2 dx}{2x^2 + 4x + 1}$.

¹see the end of <http://www.supermath.info/integrationhwk.pdf> for solutions to many of these

Problem 353 Integrate $\int \frac{(x^2 + x)dx}{x^2 - 4x + 5}$.

Problem 354 Integrate $\int \frac{(x^2 + x)dx}{x^2 - 4x + 6}$.

Problem 355 Integrate $\int \frac{x dx}{x^4 - 4x^2 + 4}$.

Problem 356 Integrate $\int \frac{x dx}{x^4 - 4x^2 + 3}$.

Problem 357 Integrate $\int \frac{(x + 1)dx}{x^4 + 4x^3 + 6x^2 + 4x + 2}$.

Problem 358 Calculate the following two rather different looking antiderivatives for the hyperbolic secant function:

(a.) Show $\int \operatorname{sech} x \, dx = \tan^{-1}(\sinh x) + C$.

Hint: $\operatorname{sech}(x) = \frac{1}{\cosh x} = \frac{\cosh x}{\cosh^2 x}$ and $\cosh^2 x = 1 + \sinh^2 x$. Try $u = \sinh x$

(b.) Show $\int \operatorname{sech} x \, dx = \sin^{-1}(\tanh x) + C$.

Hint: $\operatorname{sech}(x) = \frac{\operatorname{sech}^2(x)}{\operatorname{sech}(x)}$ and $\operatorname{sech}^2(x) = 1 - \tanh^2 x$. Try $u = \tanh x$.

Problem 359 The imaginary exponential function involves both sine and cosine:

$$e^{i\theta} = \cos \theta + i \sin \theta$$

from the above we can easily derive

$$\cos \theta = \frac{1}{2}(e^{i\theta} + e^{-i\theta}) \quad \& \quad \sin \theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$$

If $c = a + ib$ is a complex number then we likewise define

$$e^{ct} = e^{at}e^{ibt} = e^{at} \cos(bt) + ie^{at} \sin(bt).$$

Let u and v be functions of t then define $\frac{d}{dt}(u + iv) = \frac{du}{dt} + i\frac{dv}{dt}$. **Show that (3pts)**

$$\frac{d}{dt}e^{ct} = ce^{ct}$$

where $c = a + ib$ and we use the usual complex arithmetic where $i^2 = -1$. **Next, (2pts), defining $\int (u + iv)dt = \int udt + i \int vdt$, calculate $\int e^{(1+2i)t}dt$ and determine the real integrals $\int e^t \cos 2t \, dt$ and $\int e^t \sin 2t \, dt$ from examining your result.**

Problem 360 Calculate $\int \cos(ax) \cos(bx)dx$ where $a \neq b$. Hint: the mathematics in the previous problem allows calculation of a nice trigonometric identity which makes this integral simple.