Math 131, Test 3, October 28, 2008 Name: .

No graphing calculators or electronic communication of any kind. If you need extra paper please ask. Credit will be awarded for correct content and clarity of presentation. If proper notation is not present in your solution you will likely lose at least 3pts for each offense. This test has 105 points, 5 are bonus points. Make sure to at least attempt each part.

1. [12pts.] Solve the following indefinite integrals:

a.) $\int (x + e^x)dx = \frac{x^2}{2} + e^x + C$

b.) $\int \frac{1}{x}\, dx = \ln|x| + c$

c.) $\int \sec^2 x\, dx = \tan(x) + C$

d.) $\int \sqrt{x}\, dx = \frac{2}{3}x^{3/2} + C$

e.) $\int x(x^2+1)dx = \int (x^3 + x)dx = \frac{x^4}{4} + \frac{x^2}{2} + c$

f.) $\int \frac{1}{1+x^2}\, dx = \tan^{-1}(x) + C$

3.[3pts] Give the precise definition of $\int_a^b f(x)dx$ in terms of a limiting process.

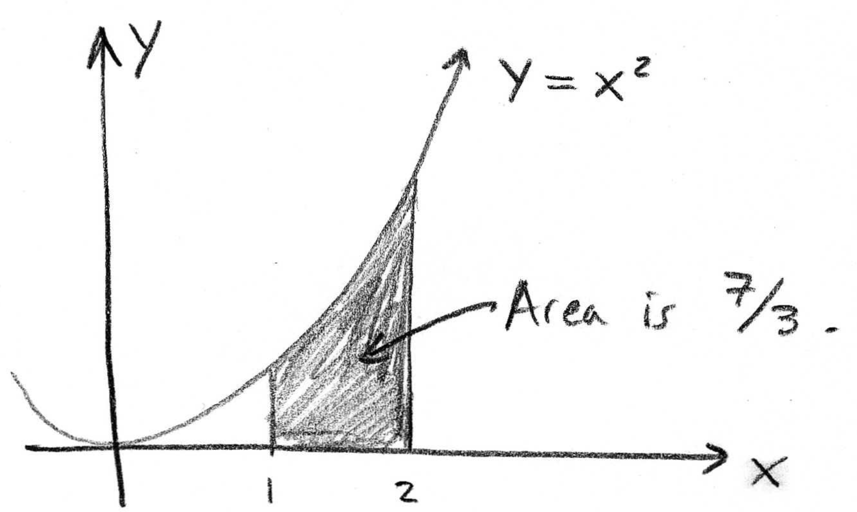
$$\int_a^b f(x)dx = \lim_{n\to\infty}\left(\sum_{i=1}^{n} f(x_i^*)\Delta x\right)$$

2. [5pts] Calculate the following definite integral.

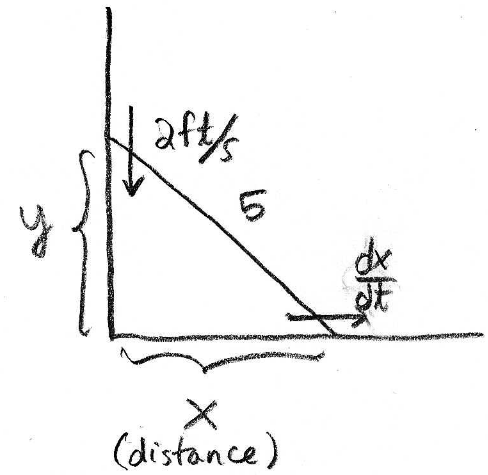
$$\int_1^2 x^2\, dx$$

Sketch a graph showing the area that the integral above describes.

$$\int_1^2 x^2 dx = \frac{1}{3}x^3\Big|_1^2 = \frac{1}{3}(8-1) = \boxed{\frac{7}{3}}$$



3. [10pts] A 5ft ladder slides down a vertical wall. If the top of the ladder is 4ft off the ground and slides downward at a rate of 2 ft/s then at what speed does the bottom of the ladder slide away from the wall?



$$x^2 + y^2 = 25$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dx}{dt} = \frac{-y}{x}\frac{dy}{dt}$$

If $y = 4$ then $x^2 + 16 = 25 \Rightarrow x = \pm 3$
but we take x to be distance so $\underline{x = 3}$
and $\frac{dy}{dt} = -2\frac{ft}{s}$ since y is decreasing.

$$\frac{dx}{dt} = \frac{-4}{3}\left(-2\frac{ft}{s}\right) = \boxed{\frac{8}{3}\frac{ft}{s}}$$

4.[40pts] Consider the function $f(x) = x^3 - 3x$. Fill in the blanks for parts a) through h). Support your answers in the space beyond the fill in the blank area. No work, no credit. Use calculus to support your conclusions. *(there are two pages provided for this problem and you may work on the back of other pages if needed, just leave me a note indicating the order of your thought process)*

a.) The function f has critical numbers at $\underline{\quad x = \pm 1 \quad}$.

b.) The function f is increasing on $\underline{\quad (-\infty, -1) \text{ and } (1, \infty) \quad}$.

c.) The function f is decreasing on $\underline{\quad (-1, 1) \quad}$.

d.) The function f has a local maximum of $\underline{\quad 2 \quad}$ at $\underline{\quad x = -1 \quad}$.

e.) The function f has a local minimum of $\underline{\quad -2 \quad}$ at $\underline{\quad x = 1 \quad}$.

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x+1)(x-1)$$

we find critical #'s $x = 1, -1$.

$$\underset{}{+\ +\ +}\ \underset{-1}{|}\ -\ -\ -\ \underset{1}{|}\ +\ +\ + \longrightarrow \frac{df}{dx}$$

Thus f is increasing on $(-\infty, -1)$ and $(1, \infty)$ while f is decreasing on $(-1, 1)$. Moreover, by 1st derivative test

$f(-1) = -1 + 3 = 2$ is local maximum.

$f(1) = 1 - 3 = -2$ is local minimum.

f.) The function f is concave up on $\underline{\quad (0, \infty) \quad}$.

g.) The function f is concave down on $\underline{\quad (-\infty, 0) \quad}$.

h.) The function has the inflection point at $\underline{(0,0)}$.

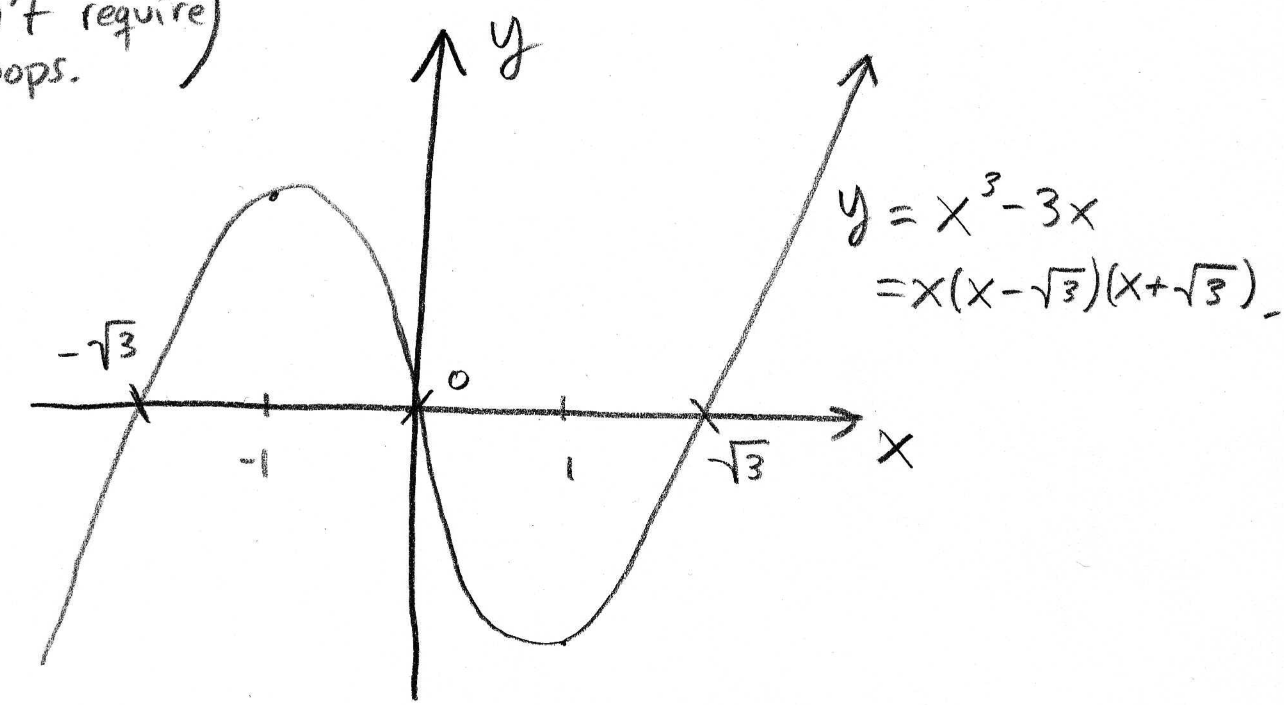
$f''(x) = 6x$ is zero for $x = 0$ only

$- \; - \; - \; - \; | \; + \; + \; + \; + \longrightarrow f''(x)$

$\qquad\qquad\quad 0$

We find f is CU on $(0, \infty)$
while f is CD on $(-\infty, 0)$.
Also $f(0) = 0$ so $\underline{(0,0) \text{ is an inflection point}}$

(I didn't require this, oops.)

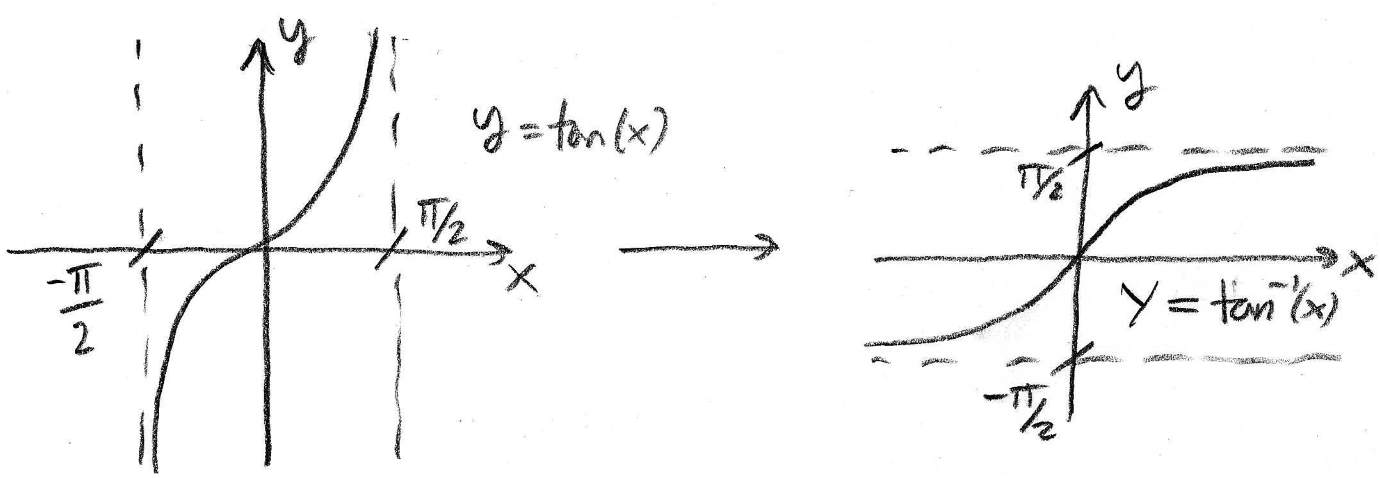


5. [15pts] Find the limits.

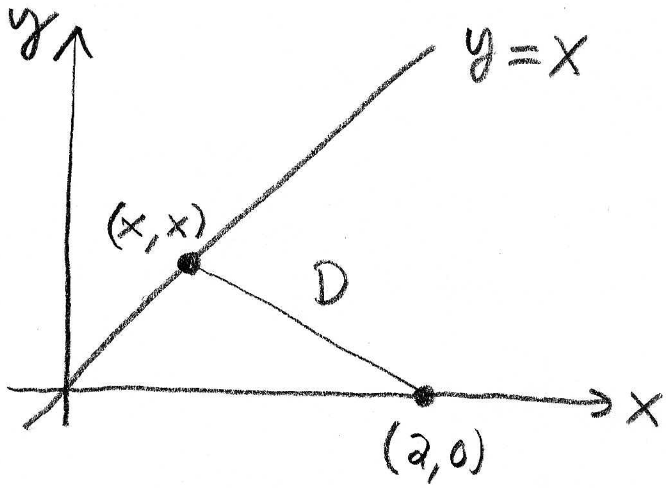
a.) $\lim_{x\to\infty} \frac{x}{2x+3} = \lim_{x\to\infty} \left(\frac{1}{2+\underbrace{3/x}_{\to 0}} \right) = \boxed{\frac{1}{2}}$

b.) $\lim_{x\to\infty} \frac{3}{2x^2+3} = \lim_{x\to\infty} \left(\frac{3/x^2}{2+3/x^2} \right) = \frac{0}{2+0} = \frac{0}{2} = \boxed{0.}$

b.) $\lim_{x\to\infty} \tan^{-1}(x) = \boxed{\frac{\pi}{2}}$



7. [15pts] Find the point on the line $y = x$ which is closest to the point $(2, 0)$. Justify your claim that the point you find is closest by an appropriate theorem from calculus.



$$D = \sqrt{(x-2)^2 + x^2}$$

$$D^2 = (x-2)^2 + x^2$$

$$2D\frac{dD}{dx} = 2(x-2) + 2x$$

$$\frac{dD}{dx} = \frac{2x-2}{D} = \frac{2}{D}(x-1) =$$

Thus the critical # is $x = 1$ since $D \neq 0$ for any x.

$[\; - \; - \; - \; | \; + \; + \; + \; + \;\to \qquad \frac{dD}{dx} = \frac{2}{D}(x-1)$

$0 \qquad\qquad 1$

Thus by 1st Derivative test $x=1$ gives D a minimum.

$\boxed{\text{The point } (1,1) \text{ is closest}}$

Alternatively: Note $\frac{dD}{dx}(1) = 0$ and $D(1) = \sqrt{1^2+1^2} = \sqrt{2}$

$$D''(x) = \frac{d}{dx}\left(\frac{2(x-1)}{\sqrt{(x-2)^2+x^2}}\right) = \frac{2D - 2(x-1)\frac{dD}{dx}}{D^2} \Rightarrow D''(1) = \frac{2}{\sqrt{2}} > 0$$

$\therefore$ Minimum By 2nd Derivative Test.

[BONUS][5pts]

a.) is it the following true or false? Explain your answer.

$$\int \sec(x)dx = \ln(\sec(x) + \tan(x)) + c$$

$$\frac{d}{dx}\left[\ln(\sec(x)+\tan(x))\right] = \frac{1}{\sec(x)+\tan(x)} \frac{d}{dx}\left(\sec(x)+\tan(x)\right)$$

$$= \frac{\sec(x)\tan(x) + \sec^2(x)}{\sec(x) + \tan(x)}$$

$$= \sec(x)\left(\frac{\tan(x) + \sec(x)}{\sec(x) + \tan(x)}\right)$$

$$= \sec(x).$$

Remark: if you complained that I should say

$$\int \sec(x)dx = \ln\left|\sec(x) + \tan(x)\right| + C$$

then I ~~was~~ would be very happy. in principle

b.) Give a, b (may not be fixed numbers) such that the following equation holds true (here is the arbitrary integration constant needed for the indefinite integral):

$$\int f(x)dx = \int_a^b f(u)du + c$$

Choose $\alpha_0 \in \mathrm{dom}(f)$. Let $x \in \mathrm{dom}(f)$

then, let F be a particular antiderivative of f

$$\frac{d}{dx}\left(\int_{\alpha_0}^{x} f(u)du\right) = \frac{d}{dx}\left(F(x) - F(\alpha_0)\right) = \frac{dF}{dx} = f(x)$$

But,

$$\frac{d}{dx}\left(\int f(x)dx\right) = f(x)$$ Recall Thm $f'(x) = g'(x) \Rightarrow f(x) = g(x) + c$

Thus $\int f(x)dx$ and $\int_{\alpha_0}^{x} f(u)du$ differ by at most a constant.

As desired.