

Homework 1 : CALCULUS II : § 7.6 # 63, 64 (STEWART 6th Ed.)

§ 7.6 # 63

$$\begin{aligned}\int \frac{1+x}{1+x^2} dx &= \int \frac{dx}{1+x^2} + \int \frac{x dx}{1+x^2} \\ &= \tan^{-1}(x) + \int \frac{\frac{1}{2} du}{u} \quad \left(\begin{array}{l} \text{letting } u = x^2 + 1 \\ \text{so } du = 2x dx \end{array} \right) \\ &= \tan^{-1}(x) + \frac{1}{2} \ln |u| + C \\ &= \tan^{-1}(x) + \frac{1}{2} \ln |x^2 + 1| + C \\ &= \boxed{\tan^{-1}(x) + \ln \sqrt{x^2 + 1} + C} \quad \left\{ \begin{array}{l} \text{noted } x^2 + 1 > 0 \\ \text{thus } |x^2 + 1| = x^2 + 1 \end{array} \right.\end{aligned}$$

§ 7.6 # 64

$$\begin{aligned}\int_0^{\pi/2} \frac{\sin x dx}{1 + \cos^2 x} &= \int_1^0 \frac{-du}{1+u^2} \quad \leftarrow \begin{array}{l} u = \cos(x) \quad du = -\sin x dx \\ u(\pi/2) = \cos \pi/2 = 0 \\ u(0) = \cos 0 = 1 \end{array} \\ &= \int_0^1 \frac{du}{1+u^2} \\ &= \tan^{-1}(u) \Big|_0^1 \\ &= \tan^{-1}(1) - \tan^{-1}(0) \\ &= \boxed{\pi/4}\end{aligned}$$