Homework 1 : Calculus II : §7.6 #63, 64 (Stewart 6th Ed.)

§7.6 #63

$$\int \frac{1+x}{1+x^2}\,dx = \int \frac{dx}{1+x^2} + \int \frac{x\,dx}{1+x^2}$$

$$= \tan^{-1}(x) + \int \frac{\frac{1}{2}du}{u} \quad \left(\text{letting } u = x^2+1 \text{ so } du = 2x\,dx\right)$$

$$= \tan^{-1}(x) + \tfrac{1}{2}\ln|u| + C$$

$$= \tan^{-1}(x) + \tfrac{1}{2}\ln|x^2+1| + C$$

$$= \boxed{\tan^{-1}(x) + \ln\sqrt{x^2+1} + C}$$

(noted $x^2+1 > 0$ thus $|x^2+1| = x^2+1$)

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§7.6 #64

$$\int_0^{\pi/2} \frac{\sin x\,dx}{1+\cos^2 x} = \int_1^0 \frac{-du}{1+u^2}$$

$u = \cos(x)$ $\quad du = -\sin x\,dx$

$u(\pi/2) = \cos \pi/2 = 0$

$u(0) = \cos 0 = 1$

$$= \int_0^1 \frac{du}{1+u^2}$$

$$= \tan^{-1}(u)\Big|_0^1$$

$$= \tan^{-1}(1) - \tan^{-1}(0)$$

$$= \boxed{\pi/4}$$