Homework 13: Calculus II: §10.4 #7, 8 (Stewart 6th Ed.)

§10.4#7 Consider population of 1000 people.

Let R = # who have heard rumor

N = # who haven't heard rumor

y = fraction who've heard rumor $= \frac{R}{1000}$

We have $R + N = 1000$ and $R = 1000y$.

$$\frac{dR}{dt} = k\left(\frac{R}{1000}\right)\left(\frac{1000-R}{1000}\right)$$

$$\Rightarrow 1000\frac{dy}{dt} = k y(1-y)$$

$$\Rightarrow \frac{dy}{dt} = My(1-y) \quad \text{(answer to part a.)}$$

(b.) Solve it.

Recall (or observe) $\frac{1}{y(1-y)} = \frac{1}{y-y^2} = \frac{1-y+y}{y(1-y)} = \frac{1}{y} + \frac{1}{1-y}$

$$\int \frac{dy}{y(1-y)} = \int M\,dt \longrightarrow \int\left(\frac{1}{y} - \frac{1}{y-1}\right)dy = \int M\,dt$$

$$\ln|y| - \ln|y-1| = Mt + C_1$$

$$\ln\left|\frac{y}{y-1}\right| = Mt + C_1$$

$$\ln\left|\frac{y-1}{y}\right| = -Mt - C_1$$

$$1 - \frac{1}{y} = ke^{-Mt}$$

$$\therefore \boxed{y = \frac{1}{1-ke^{-Mt}}}$$

§10.4 # 7 Continued

At 8AM, 80 people have heard rumor
At noon, 500 people have heard the rumor
At what time will 900 people have heard rumor?

Let $t=0$ be 8AM and $t=4$ be noon.

$$y(0) = \frac{80}{1000} = \frac{1}{1-k} \Rightarrow 1000 = 80 - 80k$$
$$\therefore k = \frac{-920}{80} = -11.5 = k$$

$$y(4) = \frac{500}{1000} = \frac{1}{1-ke^{-4M}}$$

$$2 = 1 + 11.5e^{-4M} \Rightarrow \frac{1}{11.5} = e^{-4M}$$
$$\Rightarrow 11.5 = e^{4M}$$
$$\Rightarrow M = \frac{1}{4}\ln(11.5)$$
$$\Rightarrow M \cong 0.6106$$

$$y(t) = \frac{1}{1 + 11.5e^{-0.6106t}}$$

Find t s.t. $y(t) = \frac{900}{1000} = \frac{1}{1+11.5e^{-0.6106t}}$

$$1000 = 900 + 900(11.5)e^{-0.6106t}$$
$$100 = 900(11.5)\exp(-0.6106t)$$
$$\frac{1}{9(11.5)} = \frac{1}{\exp(0.6106t)}$$
$$t = \frac{1}{0.6106}\ln(9(11.5)) \cong 7.5984$$

$t =$ 8AM + 7.598 Hours

$\longrightarrow$ **t = 3:35:24 PM**

(I leave §10.4 # 8 to you to consider)