

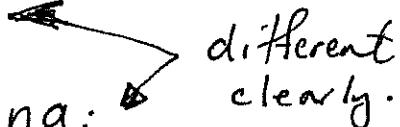
HOMEWORK 17 : CALCULUS II: §12.2 # 10, 15, 31, 34, 42, 48, 56, 60, 67, 68, 70

①

§12.2 #10

a.) $\sum_{i=1}^n a_i = a_1 + a_2 + a_3 + \dots + a_n = \sum_{j=1}^n a_j$ (no difference except the table.)

b.) $\sum_{i=1}^n a_i = a_1 + a_2 + \dots + a_n$

whereas $\sum_{i=1}^n a_j = \underbrace{a_j + a_j + \dots + a_j}_{n\text{-times}} = n a_j$ 

§12.2 #15

$$\sum_{n=1}^{\infty} 6(0.9)^{n-1} = \frac{a}{1-r} = \frac{6}{1-0.9} = [60] \quad \begin{array}{l} \text{by geometric series} \\ \text{result with } a=6 \\ \text{and } r=0.9. \end{array}$$

§12.2 #31

$$\sum_{n=1}^{\infty} \frac{\arctan(n)}{\tan^{-1}(n)}, \text{ notice } \lim_{n \rightarrow \infty} (\tan^{-1}(x)) = \frac{\pi}{2} \neq 0$$

∴ By n^{th} term test this series diverges.

§12.2 #34

$$\sum_{n=1}^{\infty} \frac{e^n}{n^2}, \text{ notice } \lim_{n \rightarrow \infty} \frac{e^n}{n^2} \neq \lim_{n \rightarrow \infty} \left(\frac{e^n}{2n} \right) \neq \lim_{n \rightarrow \infty} \left(\frac{e^n}{2} \right) = \infty \neq 0$$

↑
(extending n to be real, continuous, variable)

∴ By n^{th} term test this series diverges.

§12.2 #42

$$\begin{aligned} 0.\overline{73} &= 0.737373\dots \\ &= 0.73 + \frac{1}{100}(0.73) + \left(\frac{1}{100}\right)^2(0.73) + \dots \\ &= \frac{0.73}{1 - \frac{1}{100}} \quad \text{geom. with } a = 0.73 \text{ & } r = \frac{1}{100} \\ &= \frac{73/100}{99/100} = \boxed{\frac{73}{99}} \end{aligned}$$

§12.2 #48

(2)

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{x^n}{3^n} &= \sum_{n=1}^{\infty} \frac{x}{3} \left(\frac{x}{3}\right)^{n-1} \\ &= \frac{x/3}{1 - x/3} \quad \text{for } \left|\frac{x}{3}\right| < 1 \quad \text{by geom. series} \\ &\quad \text{result applied to } a = x/3 \text{ and} \\ &\quad r = x/3 \\ &= \boxed{\frac{x}{3-x}} \quad \leftarrow \text{for } |x| < 3 \text{ or in other words } -3 < x < 3. \end{aligned}$$

§12.2 #56

Given $\sum_{n=1}^{\infty} a_n$ has n^{th} partial sum $S_n = 3 - n2^{-n}$

we find $\sum_{n=1}^{\infty} a_n = \lim_{n \rightarrow \infty} (3 - n2^{-n}) = 3 - \underbrace{\lim_{n \rightarrow \infty} \left(\frac{n}{2^n}\right)}_{\text{goes to zero by l'Hopital's rule applied to this function real extension.}} = \boxed{3}$

To find a_n notice that

$$S_n = a_n + S_{n-1}$$

$$\therefore a_n = S_n - S_{n-1}$$

$$\Rightarrow a_n = (3 - n2^{-n}) - (3 - (n-1)2^{-(n-1)})$$

$$\Rightarrow \boxed{a_n = (n-1)2^{1-n} - n2^{-n}}$$

§12.2 #60 Find c such that $\sum_{n=0}^{\infty} e^{nc} = 10.$

$$\sum_{n=0}^{\infty} e^{nc} = 1 + e^{nc} + (e^{nc})^2 + \dots \quad \text{is geom. with } a = 1, r = e^c$$

$$\text{We want } 10 = \frac{1}{1 - e^c}$$

$$10 - 10e^c = 1 \rightarrow e^c = \frac{9}{10}$$

$$\rightarrow \boxed{c = \ln(9/10)}$$