HOMEWORK 21 : § 12.6 # 2, 3, 6, 12, 14, 18, 29, 38 : CALCULUS II STEWART 6th Ed

§ 12.6 # 2) (absolute / cond) conv or div ?

$$S = \sum_{n=1}^{\infty} \frac{n^2}{2^n}$$ use ratio test.

$$L = \lim_{n\to\infty} \left| \frac{(n+1)^2}{2^{n+1}} \frac{2^n}{n^2} \right| = \lim_{n\to\infty} \left| \left(\frac{n+1}{n}\right)^2 \frac{1}{2} \right| = \frac{1}{2} < 1$$

$\therefore$ S absolutely convergent by ratio test.

§12.6 # 3)

$$S = \sum_{n=0}^{\infty} \frac{(-10)^n}{n!} \longrightarrow L = \lim_{n\to\infty} \left| \frac{(-1)^{n+1} 10^{n+1}}{(n+1)!} \frac{n!}{(-1)^n 10^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\cancel{10^n} \cdot 10 \cdot \cancel{n!}}{(n+1)\cancel{n!}\,\cancel{10^n}} \right| = \lim_{n\to\infty} \left(\frac{10}{n+1}\right) = 0$$

$\therefore$ S is absolutely convergent by ratio test

§12.6 # 6)

$$S = \sum \frac{(-1)^n}{n^4} \Rightarrow \text{"}|S|\text{"} = \sum_{n=1}^{\infty} \frac{1}{n^4}$$ conv by $p = 4$ series test. However, this doesn't help !

$$L = \lim_{n\to\infty} \left| \frac{(-1)^{n+1}}{(n+1)^4} \frac{n^4}{(-1)^n} \right| = \lim_{n\to\infty} \left(\frac{n^4}{(n+1)^4} \right) = 1$$ well, the ratio test doesn't help either.

ratio test inconclusive here !!!

oh, Alternating Series Test! $b_n = \frac{1}{n^4}$ which is clearly decreasing, positive, and $\lim_{n\to\infty} \left(\frac{1}{n^4}\right) = 0$

$\therefore$ S converges by A.S.T.

Moreover, I argued that $\sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n^4} \right|$ converges by $P=4$ test

$\therefore$ S is absolutely convergent

§12.6 # 12

$$\sum_{n=1}^{\infty} \frac{\sin(4n)}{4^n} \longrightarrow L = \lim_{n\to\infty} \left| \frac{\sin(4n+4)}{4^{n+1}} \; \frac{4^n}{\sin 4n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\sin(4n+4)}{\sin(4n)} \; \frac{1}{4} \right|$$

d.n.e. due to oscillation $\Rightarrow$ ratio test no help.

Notice $-1 \leq \sin 4n \leq 1$

$$\Rightarrow \frac{-1}{4^n} \leq \frac{\sin 4n}{4^n} \leq \frac{1}{4^n}$$

also $0 \leq \left| \frac{\sin 4n}{4^n} \right| \leq \frac{1}{4^n}$

Note $\sum_{n=1}^{\infty} \frac{1}{4^n} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots = \frac{1/4}{1 - 1/4} = \frac{1/4}{3/4} = \frac{1}{3}$

it converges by geometric series result. Hence the D.C.T. $\Rightarrow \sum_{n=1}^{\infty} \left| \frac{\sin 4n}{4^n} \right|$ converges.

Hence by the Absolute Convergence Th$^{\underline{m}}$ $\Rightarrow \underline{\sum_{n=1}^{\infty} \frac{\sin 4n}{4^n} \text{ converges}}$.

(in fact it is absolutely convergent!)

§12.6 # 14

$$S = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2 2^n}{n!}$$

$$L = \lim_{n\to\infty} \left(\frac{(n+1)^2 2^{n+1}}{(n+1)!} \; \frac{n!}{n^2 2^n} \right)$$

$$= \lim_{n\to\infty} \left(\left(\frac{n+1}{n}\right)^2 \frac{2\cancel{n!}\,\cancel{2^n}}{(n+1)\cancel{n!}\,\cancel{2^n}} \right)$$

$$= \lim_{n\to\infty} \left(\left(\frac{n+1}{n}\right)^2 \frac{2}{n+1} \right)$$

$$= \lim_{n\to\infty} \left(\frac{2(n+1)^2}{n^2(n+1)} \right) = 0 \quad \therefore \; S \text{ is absolutely convergent.}$$

(By ratio test.)

§12.6 # 18

$$S = \sum_{n=1}^{\infty} \frac{n!}{n^n} \longrightarrow L = \lim_{n\to\infty}\left(\frac{(n+1)!}{(n+1)^{n+1}} \quad \frac{n^n}{n!}\right)$$

$$= \lim_{n\to\infty}\left(\frac{\cancel{(n+1)}\cancel{n!}\, n^n}{\cancel{(n+1)}(n+1)^n\, \cancel{n!}}\right)$$

$$= \lim_{n\to\infty}\left(\left(\frac{n}{n+1}\right)^n\right)$$

$$= \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n}$$

(look at 1^∞-type L'Hopital problems to understand this)

$$= \frac{1}{e} < 1 \quad \therefore \; S \text{ converges by ratio test}$$

(absolutely converges in fact)

§12.6 # 29

$$a_1 = 2 \quad , \quad a_{n+1} = \frac{5n+1}{4n+3}\, a_n$$

determine if $\sum_{n=1}^{\infty} a_n$ converges / diverges.

Try ratio test!

$$L = \lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\left(\frac{5n+1}{4n+3}\right)\frac{a_n}{a_n}\right|$$

$$= \lim_{n\to\infty}\left(\frac{5n+1}{4n+3}\right)$$

$$= \frac{5}{4} > 1 \quad \therefore \; \sum_{n=1}^{\infty} a_n \text{ diverges by ratio test.}$$

§12.6 # 38

Apply ratio test to obtain

$L < 1 \quad \therefore$ series converges.