§12.8 #4 Find the I.O.C & R.O.C. for $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$

Apply ratio test,

$$L = \lim_{n\to\infty} \left| \left(\frac{(-1)^{n+1} x^{n+1}}{n+2}\right)\left(\frac{n+1}{(-1)^n x^n}\right)\right| = \lim_{n\to\infty} \left| x \left(\frac{n+1}{n+2}\right)\right| = |x| \lim_{n\to\infty}\left(\frac{n+1}{n+2}\right) = |x|$$

Thus $L < 1$ iff $|x| < 1$ $\therefore$ $(-1,1) \subset$ I.O.C. Now check endpts.

$x=1$) $f(1) = \sum_{n=0}^{\infty} \frac{(-1)^n 1^n}{n+1} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$ converges by A.S.T.

$x=-1$) $f(-1) = \sum_{n=0}^{\infty} \frac{(-1)^n (-1)^n}{n+1} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots$ diverges by $p=1$ series test (harmonic series)

Therefore, $\boxed{\text{I.O.C.} = (-1, 1] \ \ \& \ \ \text{R.O.C.} = 1}$

§12.8 #6 Find I.O.C & R.O.C. for $\sum_{n=1}^{\infty} \sqrt{n}\, x^n$

Apply ratio test,

$$L = \lim_{n\to\infty} \left| \frac{\sqrt{n+1}\, x^{n+1}}{\sqrt{n}\, x^n} \right| = \lim_{n\to\infty}\left[\left(\sqrt{\frac{n+1}{n}}\right)|x|\right] = |x| \lim_{n\to\infty} \sqrt{1+\frac{1}{n}} = |x|.$$

Hence $L < 1$ iff $|x| < 1$ $\therefore$ $(-1,1) \subset$ I.O.C. & R.O.C. = 1.

Check end pts.

$x=1$) $\sum_{n=1}^{\infty} \sqrt{n}$ diverges by n^{th} term test $\lim_{n\to\infty} \sqrt{n} \neq 0$

$x=-1$) $\sum_{n=1}^{\infty} \sqrt{n}(-1)^n$ diverges by n^{th} term test $\lim_{n\to\infty} (\sqrt{n})(-1)^n \neq 0$

Hence, $\boxed{\text{I.O.C.} = (-1, 1) \ \ \& \ \ \text{R.O.C.} = 1}$

§12.8 #12 $\sum_{n=1}^{\infty} \frac{x^n}{5^n n^5}$ find the I.O.C. & R.O.C.

As usual, $L = \lim_{n\to\infty} \left| \frac{x^{n+1}}{5^{n+1}(n+1)^5} \frac{5^n n^5}{x^n} \right| = \lim_{n\to\infty} \left[|x| \cdot \frac{n^5}{(n+1)^5} \cdot \frac{1}{5} \right] = \frac{|x|}{5}$.

Thus $L < 1$ iff $|x|/5 < 1 \Leftrightarrow |x| < 5$

Hence $(-5,5) \subset$ I.O.C. & R.O.C. = 5. Check Endpts. $x = \pm 5$.

$x=5$) $\sum_{n=1}^{\infty} \frac{1}{n^5}$ converges by $p=5$ test. $x=-5$) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^5}$ converges by A.S.T. test

Therefore, $\boxed{\text{I.O.C.} = [-5, 5] \ \ \& \ \ \text{R.O.C.} = 5}$

$\boxed{\S 12.8 \# 25}$ Find I.O.C & R.O.C. for $\displaystyle\sum_{n=1}^{\infty} \frac{(4x+1)^n}{n^2}$

Notice $\displaystyle L = \lim_{n\to\infty} \left| \frac{(4x+1)^{n+1}}{(n+1)^2} \frac{n^2}{(4x+1)^n} \right|$

$$= \lim_{n\to\infty} \left[|4x+1| \left(\frac{n}{n+1}\right)^2 \right]$$

$$= |4x+1| < 1 \Rightarrow -1 < 4x+1 < 1$$

$$\Rightarrow -2 < 4x < 0$$

$$\Rightarrow -\tfrac{1}{2} < x < 0 \quad \therefore \underline{(-1/2, 0) \subset \text{I.O.C.}}$$

Check Endpts,

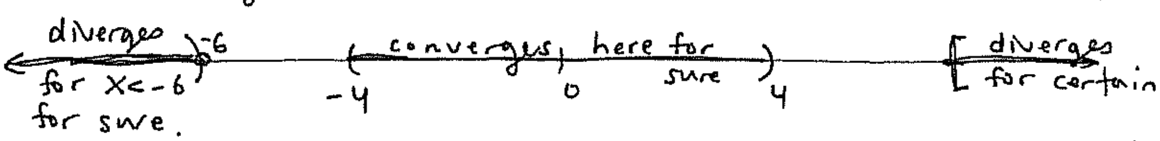
$\underline{x=0}$ $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ conv. by $p=2$

$\underline{x=-\frac{1}{2}}$ $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ conv. by A.S.T.

Thus $\boxed{\text{I.O.C.} = [-1/2, 0] \quad \text{R.O.C.} = 1/4}$

$\boxed{\S 12.8 \# 30}$ Suppose $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ converges when $x=-4$ and diverges for $x=6$. What can we say about (a.) $\displaystyle\sum_{n=0}^{\infty} c_n$, (b.) $\displaystyle\sum_{n=0}^{\infty} c_n 8^n$, (c.) $\displaystyle\sum_{n=0}^{\infty} c_n (-3)^n$ (d.) $\displaystyle\sum_{n=0}^{\infty} (-1)^n c_n 9^n$

To answer these we note that $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ is centered at $a=0$ thus convergence at $x=-4 \Rightarrow$ I.O.C. $\underline{\text{includes}}$ $(-4, 4)$



Between $-6 \le x \le -4$ and $4 \le x < 6$ we're ignorant. Hence,

(a.) $\displaystyle\sum_{n=0}^{\infty} c_n$ converges.

(b.) $\displaystyle\sum_{n=0}^{\infty} c_n 8^n$ diverges

(c.) $\displaystyle\sum_{n=0}^{\infty} c_n (-3)^n$ converges

(d.) $\displaystyle\sum_{n=0}^{\infty} c_n (-9)^n$ diverges.

All of these based on the I.O.C. $\text{Th}^{\underline{m}}$ which says the I.O.C. is either $\mathbb{R}$, a (half)(open)(closed) interval or just the center.

§12.8 #32 Let $p, q \in \mathbb{R}$ such that $p < q$. Find power series whose I.O.C. is
(a.) (p,q), (b.) $(p,q]$, (c.) $[p,q)$ (d.) $[p,q]$

To begin notice we want our center at the <u>midpoint</u> thus we want $a = \frac{1}{2}(p+q)$. Thus

$$\boxed{f(x) = \sum_{n=0}^{\infty} c_n \left(x - \frac{p+q}{2}\right)^n}$$

Then use, for example,

(a.) $c_n = 1$ (endpts fail due to n^{th} term test)

(b.) $c_n = \frac{(-1)^n}{n}$ (left endpt harmonic, right A.S.T.)

(c.) $c_n = \frac{(-1)^{n+1}}{n}$ (left endpt. alt. harmonic, right harmonic)

(d.) $c_n = \frac{1}{n^2}$ (A.S.T. on left endpt, $p=2$ on right endpt.)

§12.8 #38 If $f(x) = \sum_{n=0}^{\infty} c_n x^n$ where $c_{n+4} = c_n$ for all $n \geq 0$ then find the I.O.C. and find a nice f-la for $f(x)$

Notice $c_0 = c_4 = c_8 = \cdots = c_{4n}$, $c_1 = c_5 = \cdots = c_{4n+1}$, $c_2 = c_6 = \cdots = c_{4n+2}$ and $c_3 = c_7 = c_{11} = \cdots = c_{4n+3}$ thus the series breaks into 4 parts,

$$f(x) = \sum_{n=0}^{\infty} c_{4n} x^{4n} + \sum_{n=0}^{\infty} c_{4n+1} x^{4n+1} + \sum_{n=0}^{\infty} c_{4n+2} x^{4n+2} + \sum_{n=0}^{\infty} c_{4n+3} x^{4n+3} \quad \star$$

$$= c_0 \sum_{n=0}^{\infty} x^{4n} + x c_1 \sum_{n=0}^{\infty} x^{4n} + x^2 c_2 \sum_{n=0}^{\infty} x^{4n} + x^3 c_3 \sum_{n=0}^{\infty} x^{4n}$$

$$= \underbrace{\boxed{\frac{c_0 + c_1 x + c_2 x^2 + c_3 x^3}{1 - x^4}}}_{\text{"nice" formula.}}$$

identifying $a = 1$ and $r = x^4$ for each summation above

$\therefore |x^4| < 1 \Rightarrow \boxed{(-1,1) = \text{I.O.C}}$

Question: why is it ok to rearrange the power series as I did in step $\star$?

§12.8 #41 Suppose $\sum c_n x^n$ has R.O.C $= 2$ & $\sum d_n x^n$ has R.O.C. $= 3$ what is the R.O.C. of $\sum (c_n + d_n) x^n$?

$$\text{dom}(f+g) = \text{dom}(f) \cap \text{dom}(g)$$

$$\Rightarrow (-2,2) \cap (-3,3) = (-2,2) \cdots \boxed{\text{R.O.C.} = 2}$$