HOMEWORK 25 : §12.11 # 4, 8, 28 // STEWART 6th Ed. (1)

§12.11 #4 Find the $T_3(x)$ for $f(x) = x + e^{-x}$ centered at $a = 0$

Note $f'(x) = 1 - e^{-x}$ and $f''(x) = e^{-x}$ and $f'''(x) = -e^{-x}$ hence,

$$T_3(x) = f(0) + f'(0)x + \tfrac{1}{2}f''(0)x^2 + \tfrac{1}{6}f'''(0)x^3$$

$$= \boxed{1 + \tfrac{1}{2}x^2 + \tfrac{1}{6}x^3}$$

(I leave the graphing for the reader)

§12.11#8 Calculate $T_3(x)$ at $a = 1$ for $f(x) = \frac{1}{x}\ln(x)$

$$f'(x) = \frac{-1}{x^2}\ln(x) + \frac{1}{x^2} = \frac{1}{x^2}(1 - \ln(x)) \quad \therefore \quad \underline{f'(1) = 1.}$$

$$f''(x) = \frac{-2}{x^3}(1 - \ln(x)) - \frac{1}{x^3} = \frac{1}{x^3}(2\ln(x) - 3) \quad \therefore \quad \underline{f''(1) = -3.}$$

$$f'''(x) = \frac{-3}{x^4}(2\ln(x) - 3) + \frac{2}{x^4} = \frac{1}{x^4}(11 - 6\ln(x)) \quad \therefore \quad \underline{f'''(1) = 11.}$$

Hence,

$$T_3(x) = f(1) + f'(1)(x-1) + \tfrac{1}{2}f''(1)(x-1)^2 + \tfrac{1}{6}f'''(1)(x-1)^3$$

$$= \boxed{(x-1) - \tfrac{3}{2}(x-1)^2 + \tfrac{11}{6}(x-1)^3}$$

§12.11 #28 What range of x is $\cos(x) \approx 1 - \frac{x^2}{2} + \frac{x^4}{24}$ correct to within a margin of error of 0.005 ?

The alt. series. estimation Thm indicates that for each $x \in \mathbb{R}$ we have $\left|\cos(x) - \left(1 - \frac{x^2}{2} + \frac{x^4}{24}\right)\right| < \left|\frac{x^6}{6!}\right|$ since $\left|\frac{x^6}{6!}\right|$ is the <u>next</u> term beyond those in the actual approximating fnct.

We want $\left|\frac{x^6}{6!}\right| \leq 0.005$ (or better $\frac{x^6}{6!} < 0.005$ to be safe.)

$$x^6 \leq (0.005)\,6\cdot 5\cdot 4\cdot 3\cdot 2$$

$$|x| \leq 3.6 \quad \therefore \quad \boxed{-3.6 \leq x \leq 3.6}$$

(radians!)