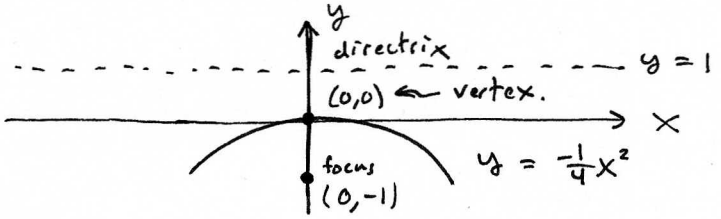
HOMEWORK 27 : CALCULUS II : §11.5 # 2, 4, 12, 16, 20, 24, 26, 30, 40, 48, 55

§11.5#2 Find vertex, focus and directrix of parabola and sketch graph for $4y + x^2 = 0$

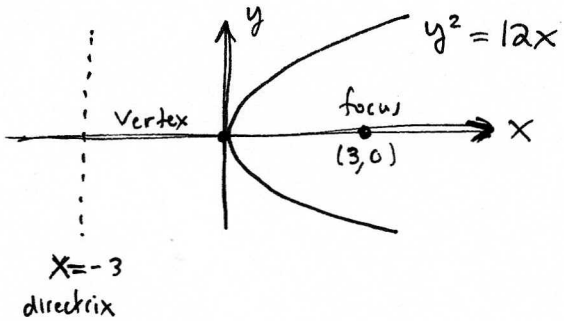
Eqⁿ (1) states a parabola with focus $(0, p)$ and directrix $y = -p$ is $x^2 = 4py$. We have $x^2 = -4y \Rightarrow \underline{p = -1}$ hence

focus = $(0, -1)$ and directrix; $y = 1$



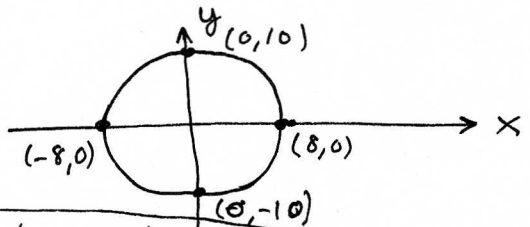
§11.5#4 Find vertex, directrix & focus for $y^2 = 12x$

Notice $y^2 = 4(3)x$ thus $p = 3 \Rightarrow$ vertex at $(0,0)$, directrix $x = -3$ and focus at $(3, 0)$ (just switch roles of x & y in previous problem)



§11.5#12 I'll sketch graph of $\frac{x^2}{64} + \frac{y^2}{100} = 1$

I usually let $x = 0 \Rightarrow y^2 = 100 \Rightarrow y = \pm 10 \Rightarrow (0, \pm 10)$ on graph.
Also $y = 0 \Rightarrow x^2 = 64 \Rightarrow x = \pm 8 \Rightarrow (\pm 8, 0)$ also on graph.



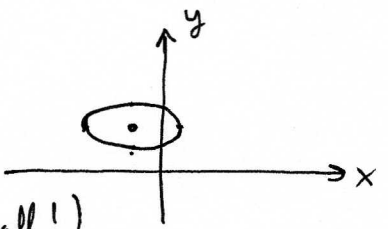
§11.5#16 Sketch graph of $x^2 + 3y^2 + 2x - 12y + 10 = 0$

Complete squares to locate the center of the ellipse,

$$x^2 + 2x + 3y^2 - 12y + 10 = 0$$

$$\Rightarrow (x+1)^2 + 3(y-2)^2 = 1 + 12 - 10 = 3$$

centered at $(-1, 2)$



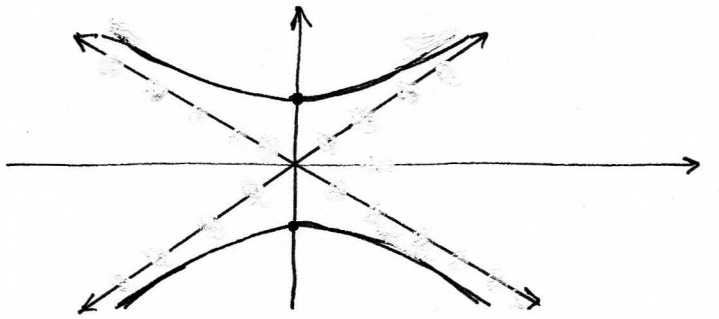
(my graph too small!)

§11.5 #20 Sketch graph of hyperbola $y^2/16 - x^2/36 = 1$

We need two things to graph hyperbola nicely; asymptotes and knowledge of which way it opens. Notice $y=0 \Rightarrow x^2 = -36$ hence $y=0$ is not contained in graph. In contrast $x=0 \Rightarrow y^2 = 16 \Rightarrow y = \pm 4$ $\therefore$ $(0,4)$, $(0,-4)$ on hyperbola.

Asymptotes concern $x \gg 1$ where $y^2/16 \approx x^2/36$ thus,

$y^2 \approx \frac{16}{36}x^2 \Rightarrow \underline{y = \pm \frac{4}{6}x}$ asymptotes

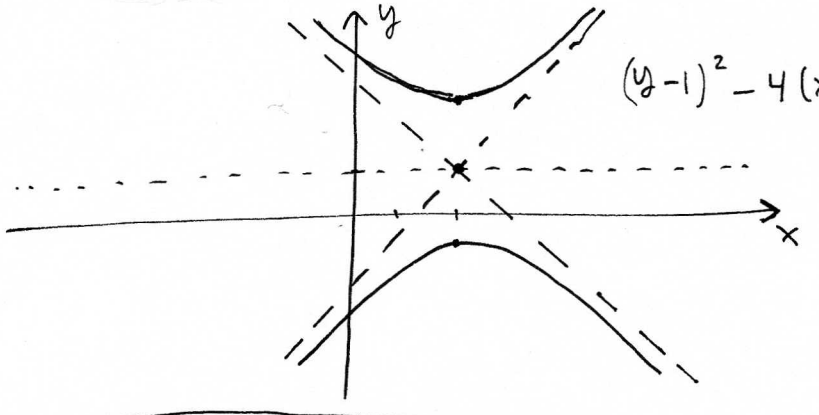


§11.5 #24 Graph $y^2 - 4x^2 - 2y + 16x = 31$

We complete square to identify the center:

$y^2 - 4x^2 - 2y + 16x = 31 \Rightarrow (y-1)^2 - 4(x-2)^2 = 31 + 1 - 16 = 16$

Thus $\frac{(y-1)^2}{16} - \frac{(x-2)^2}{4} = 1$ centered at $(2,1)$ and it opens vertically (note $y=1 \Rightarrow -(x-2)^2 = 4 \Rightarrow$ no sol^n)



§11.5 #26 $x^2 = y^2 + 1$ identify the conic

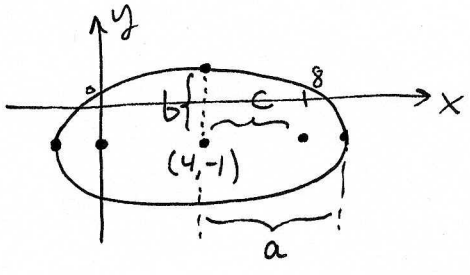
Notice $x^2 - y^2 = 1$ this is a <u>hyperbola</u>. It does not cross $x=0$ hence it opens <u>horizontally</u>.

§11.5#30 Identify the conic $4x^2 + 4x + y^2 = 0$

$4(x^2+1) + y^2 = 0$

$\Rightarrow 4(x+\frac{1}{2})^2 + y^2 = 1$ $\therefore$ ellipse centered at $(-1/2, 0)$.

§11.5#40 Find eq^n of an ellipse with focal pts $(0,-1)$, $(8,-1)$ and vertex $(9,-1)$.



symmetry suggests $(-1,-1)$ also vertex
then the center is clearly $(4,-1)$.
Eq^n (4) says $c^2 = a^2 - b^2$.
It appears $c = 4$ and $a = 5$
then $16 = 25 - b^2 \Rightarrow b^2 = 9 \Rightarrow b = \pm 3$.

Thus, $\boxed{\frac{(x-4)^2}{(5)^2} + \frac{(y+1)^2}{(3)^2} = 1}$

– I leave #48 & #55 to the reader. My goal in assigning these was to get you to think about the graphs of ellipses and hyperbolas. I don't expect you memorize eq^{n}s for the focal pts. or directrix however I do hope you can identify the <u>type</u> of conic as well as general features such as opening left/right/up/down and I do hope you can complete squares as need to identify centers.