

(1)

§ 8.1 # 3)

$$\int \underbrace{x}_u \underbrace{\cos(5x)}_{dv} dx = \frac{1}{5} x \sin(5x) - \int \frac{1}{5} \sin(5x) dx$$

$$= \boxed{\frac{1}{5} x \sin(5x) + \frac{1}{25} \cos(5x) + C}$$

§ 8.1 # 4)

$$\int \underbrace{x}_u \underbrace{e^{-x}}_{dv} dx = -x e^{-x} + \int e^{-x} dx$$

$$= \boxed{-x e^{-x} - e^{-x} + C}$$

§ 8.1 # 7)

$$\int \underbrace{x^2}_u \underbrace{\sin(\pi x)}_{dv} dx = x^2 \left(\frac{-1}{\pi} \cos \pi x \right) + \int \frac{1}{\pi} \cos(\pi x) 2x dx$$

$$= -\frac{x^2}{\pi} \cos \pi x + \int \frac{2x}{\pi} \underbrace{\cos(\pi x)}_{dv} dx$$

$$= -\frac{x^2}{\pi} \cos \pi x + \frac{2x}{\pi^2} \sin(\pi x) - \int \frac{2}{\pi^2} \sin(\pi x) dx$$

$$= \boxed{-\frac{x^2}{\pi} \cos \pi x + \frac{2x}{\pi^2} \sin(\pi x) + \frac{2}{\pi^3} \cos(\pi x) + C}$$

§ 8.1 # 8)

$$\int \underbrace{x^2}_u \underbrace{\cos(mx)}_{dv} dx = \frac{1}{m} x^2 \sin(mx) - \int \frac{2x}{m} \underbrace{\sin(mx)}_{dv} dx$$

$$= \frac{1}{m} x^2 \sin(mx) + \frac{2x}{m^2} \cos(mx) - \int \frac{1}{m} \cos(mx) \frac{2}{m} dx$$

$$= \boxed{\frac{x^2}{m} \sin(mx) + \frac{2x}{m^2} \cos(mx) - \frac{2}{m^3} \sin(mx) + C}$$

§ 8.1 # 10)

$$\int \underbrace{\sin^{-1}(x)}_u \underbrace{dx}_{dv} = x \sin^{-1}(x) - \int \frac{x dx}{\sqrt{1-x^2}} \quad : \quad \begin{cases} u = 1-x^2 \\ du = -2x dx \end{cases}$$

$$= x \sin^{-1}(x) - \int \frac{1}{\sqrt{u}} \left(-\frac{1}{2} du \right)$$

$$= x \sin^{-1}(x) + \sqrt{u} + C = \boxed{x \sin^{-1}(x) + \sqrt{1-x^2} + C}$$

§ 8.1 # 12)

(2)

$$\begin{aligned}\int \underbrace{p^5 \ln(p)}_u dp &= \frac{1}{6} p^6 \ln(p) - \int \frac{1}{6} p^6 \frac{dp}{p} \\ &= \frac{p^6}{6} \ln(p) - \frac{1}{6} \int p^5 dp \\ &= \boxed{\frac{1}{6} p^6 \ln(p) - \frac{1}{36} p^6 + C}\end{aligned}$$

§ 8.1 # 17)

$$\begin{aligned}\int \underbrace{e^{2\theta} \sin(3\theta)}_u d\theta &= \frac{1}{2} e^{2\theta} \sin(3\theta) - \int \frac{1}{2} e^{2\theta} \cdot 3 \cos(3\theta) d\theta \\ &= \frac{1}{2} e^{2\theta} \sin(3\theta) - \frac{3}{2} \int \underbrace{\cos(3\theta)}_u \frac{e^{2\theta} d\theta}{dv} \\ &= \frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{2} \left[\frac{1}{2} e^{2\theta} \cos 3\theta + \int \frac{3}{2} e^{2\theta} \sin(3\theta) d\theta \right] \\ &= \frac{1}{2} e^{2\theta} (\sin(3\theta) - \frac{3}{2} \cos(3\theta)) - \frac{9}{4} \int e^{2\theta} \sin 3\theta d\theta\end{aligned}$$

$$\Rightarrow \frac{13}{4} \int e^{2\theta} \sin 3\theta d\theta = \frac{1}{2} e^{2\theta} \sin 3\theta - \frac{3}{4} e^{2\theta} \cos 3\theta + C_1$$

$$\Rightarrow \boxed{\int e^{2\theta} \sin 3\theta d\theta = \frac{2}{13} e^{2\theta} \sin 3\theta - \frac{3}{13} e^{2\theta} \cos 3\theta + C}$$

§ 8.1 # 33)

$$\int \cos \sqrt{x} dx = \int \cos(t) \cdot 2t dt$$

$$\text{Let } t = \sqrt{x} \\ dt = \frac{dx}{2\sqrt{x}} \Rightarrow dx = 2t dt$$

$$= \int \underbrace{2t}_u \underbrace{\cos t}_{dv} dt$$

$$= 2t \sin t - \int \sin t \cdot 2 dt$$

$$= 2t \sin t + 2 \cos t + C$$

$$= \boxed{2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C}$$