

§8.1 #34

$$\begin{aligned}
 \int t^3 e^{-t^2} dt &= \int t^2 e^{-t^2} t dt \\
 &= \int u e^{-u} \cdot \frac{1}{2} du \quad : \quad \underline{u = t^2}, \quad du = 2t dt \\
 &= \frac{-1}{2} u e^{-u} + \frac{1}{2} \int e^{-u} du \quad : \quad \left[\begin{array}{l} \bar{u} = u \\ d\bar{v} = \frac{1}{2} e^{-u} du \\ \bar{v} = -\frac{1}{2} e^{-u} \end{array} \right] \\
 &= -\frac{1}{2} u e^{-u} - \frac{1}{2} e^{-u} + C \\
 &= \boxed{-\frac{1}{2} t^2 e^{-t^2} - \frac{1}{2} e^{-t^2} + C}
 \end{aligned}$$

 §8.1 #49 Assume $n \neq 1$ and ignore STEWART'S HINT SINCE THIS IS WAY EASIER \textcircled{U}

$$\begin{aligned}
 \int \tan^n(x) dx &= \int \tan^{n-2}(x) \tan^2(x) dx \\
 &= \int \tan^{n-2}(x) (\sec^2 x - 1) dx \\
 &= \int u^{n-2} du - \int \tan^{n-2}(x) dx \quad : \quad \begin{array}{l} u = \tan(x) \\ du = \sec^2 x dx \end{array} \\
 &= \frac{1}{n-1} \tan^{n-1}(x) - \int \tan^{n-2}(x) dx
 \end{aligned}$$

§8.1 #64

Suppose $f(0) = g(0) = 0$ and f'' & g'' are continuous,

$$\begin{aligned}
 \int_0^a f(x) \underbrace{g''(x)}_{u} dx &= f(x) g'(x) \Big|_0^a - \int_0^a \underbrace{\frac{g'(x) f'(x)}{u}}_{dV} dx \\
 &= f(x) g'(x) \Big|_0^a - f'(x) \underbrace{g(x)}_{dV} \Big|_0^a + \int_0^a g'(x) f''(x) dx \\
 &= f(a) g'(a) - f(0) g'(0) - f'(a) g(a) + f'(0) g(0) - \int_0^a g(x) f''(x) dx \\
 &= \boxed{f(a) g'(a) - f'(a) g(a) - \int_0^a g(x) f''(x) dx}.
 \end{aligned}$$