

## REVIEW FOR TEST 1 OF CALCULUS II FALL 2009:

The first and best line of defense is to complete and understand the homework and lecture examples. Past that my old test might help you get some idea of how my tests typically look (although our course differs significantly in content). Most of the test will be like problems you've done before, they may not be the same format but they should require the same skill set.

- the test is likely to divide up about 10% L'Hopital's Rule, 15% integrals of trig functions, 20% trig. subst., 20% IBP, 20% partial fractions, 10% improper integration. ( Bonus out of class.)

### A Few Reminders on Basics of Integration:

1. Antiderivatives or indefinite integration: know your basic antiderivatives. I might ask all of them. Page 197 #1-14, I assume you know these. Integrals 15-18 would be given if I expected you use them.
2. What is the FTC? Be able to apply it to definite integrals.
3. Notice there are absolute value bars in  $\int \frac{1}{x} dx = \ln |x| + c$ . They matter.
4. Know the two main methods to calculate definite integrals involving u-subst. I may ask a question which forces you to change the bounds. (see pg. 199)

### Trigonometry:

1. Either know how to use the imaginary exponentials to derive trig. identities or be able to derive them by some other means. In particular, be able to do calculations like 9.3.3.1, 9.3.3.2, 9.3.3.3, 9.3.3.4. You should memorize the following formulas so you can derive things if need be,

$$\cos(x) = \frac{1}{2} \left( e^{ix} + e^{-ix} \right) \quad \sin(x) = \frac{1}{2i} \left( e^{ix} - e^{-ix} \right)$$

2. Let me reiterate: Memorize the formulas in section 9.3.4, or learn an efficient way to derive them. Remember there are many trig. integrals that are just about impossible unless you know these formulas. ( the imaginary exponential technique gets most of those formulas pretty quickly, but you can also use Dr. Kester's idea to derive everything from the adding-angles formulas [ those are 1 and 2 on the list] )
3. Be able to integrate trig. functions we have encountered in homework and/or lecture. For example, in the cases  $n, m = 1, 2, 3, 4, \dots$  you should be able to integrate :  $\sin^n(\theta)$ ,  $\cos^n(\theta)$ ,  $\tan^n(\theta)$ ,  $\sin^m(\theta) \cos^n(\theta)$ ,  $\tan^m(\theta)$ ,  $\sec^n(\theta)$ .

### Trig. Substitution:

1. Be able to apply the three main substitutions as indicated in the box on 210.
2. Remember, you must change both the integrand and the measure (dx).
3. Know about the triangle idea. If I ask to leave your answer in a “nice algebraic form” I mean that you should avoid writing as many inverse trig functions as possible. Of course there are cases where we have a “naked theta” so we don’t have the luxury of avoiding inverse trig. functions.
4. I do not expect you to know when to make a hyperbolic substitution, if I am to ask a question about hyperbolic substitution I would give a fair amount of guidance, much like the early trig-subst. problems where they tell you the subst.

### Integration by Parts:

1. Be aware of which examples need IBP and which do not.
2. Consider using LIATE to help guide your choice of u and dv
3. Keep in mind that sometimes you might need to apply IBP several times to solve a problem (see E9.5.5)
4. Also, notice sometimes you will need to solve for the integral when the IBP loops back (see E9.5.6 and E9.5.8)
5. What happens if the given integral has bounds and yet you need to do an IBP to solve it? Can you solve the following?

$$\int_1^2 \ln(x) dx$$

### Partial Fractions:

1. Can you factor a cubic given a root? How would you factor  $x^3 + 2x^2 - 3$  if I gave you the hint that this polynomial has a zero at  $x = 1$ ? This would be an issue if I asked you to integrate  $\int \frac{dx}{x^3 + 2x^2 - 3}$ .
2. I may ask a question that requires long division or the adding zero trick. See E9.6.1 and E9.6.6. How would you integrate  $\int \frac{2x dx}{x+3}$  or  $\int \frac{x^2 dx}{x^2+9}$ ?
3. Know your algebra. Be able to completely factor reasonable polynomials. Make clear the distinction between  $x^2 + 1$ ,  $x^2 - 1$ ,  $(x + 1)^2$ . Why do we need to factor? What do we need to factor?
4. Be able to set-up the partial fractional decomposition for any rational function which has a completely factored denominator. For example, E9.6.4 and E9.6.5. Also, be able to find the coefficients  $A, B, C, \dots$  for reasonable examples. An example is reasonable if we did a similar one in lecture or homework.
5. Know your basic rational function integrals, (formulas 1-5 on page 219). I would not recommend memorizing 4 and 5, rather know how to do those integrals. There is a good chance I would ask you to do it separately so you could just use it when you got to the partial fractions problem.

### Improper Integration:

1. Know the definition of integral with a bound of  $\pm\infty$  in terms of a limit which goes to  $\pm\infty$ . Be able to calculate integrals with infinite limits for reasonable functions. That means you need to know the limits of all the basic functions, which really means you should know their graphs. Be aware you might need to apply L'hospital's rule. ( see E9.7.1 through E9.7.7 )
2. If we are asked to calculate a definite integral which includes the point where the integrand is not well-defined ( for example, if the integrand has a vertical asymptote or a hole in the graph) then we must use a limiting process to approach the trouble point. Be able to do problems like E9.7.8-9.7.12
3. Beware that it is possible to have problems which are improper in both ways. For example, E9.7.13.