This project is due Saturday May 2, 2009 by 5pm in the box outside my office. The final exam will be on Wednesday May 6, 8am in the usual room. If you have many finals on that day and you would like to take it early please let me know soon so I can accomodate you.

Please write your answers clearly beside the problem. Provide supporting calculations which justify your answers on other paper (please provide work in order with proper numbering in your supporting calculations). No justification, no credit. Partial credit is possible for completion of just part of the project, it will be awarded as indicated in the point break-down beside the problems

These problems are not comprehensive, but they ought to help. I will not provide a formal final exam review this semester (I'll answer questions, but you have to ask them on the review day which is Wednesday April 29). Sufficient to say that if it was on a test before it is certainly fair game for the final. I do expect there will be problems from each portion of the material. I expect you know the difference between a sequence and a series. I also expect you can answer basic questions which test your understanding of that distinction. Also, I expect you be fluent in all the trigonometric identities we have discussed this semester. You should be able to derive whatever you need either from the imaginary exponentials or the adding angles formulas. These bonus points will apply to the test grade.

Problem 1 [1/2pt] Integrate.

$$\int \sqrt{9-x^2}\,dx$$

.

Problem 2 [1/2pt] Integrate.

$$\int \frac{dy}{\sqrt{y^2+16}}$$

.

Problem 3 [1/2pt] Integrate.

$$\int \frac{tdt}{\sqrt{9-t^2}}$$

.

Problem 4 [1/2pt] Integrate.

$$\int_1^2 x\ln(x)\,dx$$

.

Problem 5 [1/2pt] Integrate.

$$\int \frac{dx}{(x^2+1)(2+\tan^{-1}(x))}$$

Problem 6 [1/2pt] Integrate.

$$\int \frac{dy}{y^2 - 4y + 8}$$

.

Problem 7 [1/2pt] Integrate.

$$\int \cos^2(3x)\, dx$$

.

Problem 8 [1/2pt] Integrate.

$$\int \tan^3(2t)\, dt$$

.

Problem 9 [1/2pt] Integrate.

$$\int \frac{x^2}{x^2 + 4}\, dx$$

.

Problem 10 [1/2pt] Integrate.

$$\int \ln(x+1)\, dx$$

.

Problem 11 [1/2pt] Integrate.

$$\int \frac{z + 1}{z^2(z^2 + 4)}\, dz$$

.

Problem 12 [1/2pt] Integrate.

$$\int \frac{x}{x^3 + 4x^2 + 5x}\, dx$$

.

Problem 13 [1/2pt] Integrate.

$$\int_1^\infty \frac{\ln(y)}{y^3}\, dy$$

.

Problem 14 [1/2pt] Integrate.

$$\int_0^\infty te^{-t}\, dt$$

.

Problem 15 [1pt] Integrate (warning, this one is "challenging").

$$\int \frac{dx}{x^4 + 4}$$

.

Problem 16 [2pt] Find Maclaurin series expansions for $f(x), g(x), h(\beta)$ and $j(z)$ via the known expansions or geometric series tricks:

(*a.*) $f(x) = xe^x + 3$.

(*b.*) $g(x) = 3\sin(x+4)$.

(*c.*) $h(\beta) = \dfrac{\beta}{1-\beta^2}$.

(*d.*) $j(z) = \ln(\sqrt{1+z^2})$.

Problem 17 [1.5pt] Find Maclaurin series expansions for $f(x), g(x)$ and $h(x)$ via the geometric series result and/or the FTC.

(*a.*) $f(x) = \dfrac{1}{x-2}$.

(*b.*) $g(x) = \ln(3+x^3)$.

(*c.*) $h(x) = \dfrac{1}{x^2+4x+3}$.

Problem 18 [1/2pt] Calculate the first 4 nontrivial terms in the power series expansion of $f(x) = \sqrt{e^x + \sin(x)}$ centered around $a = \pi$.

Problem 19 [2pt] Find the first three nontrivial terms for power series solutions of the integrals below.

(*a.*) $\displaystyle\int \sqrt{e^x + \sin(x)}\, dx$.

(*b.*) $\displaystyle\int \frac{1-\cos(x)}{x}\, dx$.

(c.) $\int \frac{dx}{(1+x^2)^{\frac{1}{7}}}$.

(d.) $\int \left(\frac{1}{1-x}\right) e^{x+2}\, dx$.

Problem 20 [2.5pt] Find the general solution to the differential equations given below and impose initial condition in part *b*.

(a.) $x\frac{dy}{dx} + 2y = 1 - \frac{1}{x}, \qquad x > 0$.

(b.) $\frac{dy}{dx} = \frac{\cos(x+3)}{2y}, \ y(0) = -5$

(c.) $y'' - y = 0$.

(d.) $y'' + 14y' + 24y = 0$.

(e.) $y'' + 10y' + 25y = 0$.

Problem 21 [1pt] Find the IOC and ROC for the power series below:

(a.) $\sum_{n=0}^{\infty} (2x+7)^{n+3}$.

(b.) $\sum_{n=0}^{\infty} \frac{n}{n^2+3} 3^n (2-x)^n$.

Problem 22 [1.25pt] Determine if the sequence covergences or diverges. Justify your answer if necessary (it is necessary to resolve indeterminant forms via some technique like L'Hopital's rule etc...)

(a.) $\{1, -1, 1, -1, 1, -1, \dots\}$.

(*b.*) $\{1/n\}_{n=1}^{\infty}$.

(*c.*) $\left\{\dfrac{n!}{(n+2)!}\right\}_{n=1}^{\infty}$.

(*d.*) $\left\{\dfrac{e^{-n}}{n}\right\}_{n=1}^{\infty}$.

(*e.*) $\left\{\dfrac{n}{e^{-n}}\right\}_{n=1}^{\infty}$.

Problem 23 [1.25pt] Find the three nontrivial terms in the sequence of partial sums. If possible calculate the limit of the sequence of partial sums, otherwise explain why the series converges or diverges via one of the convergence tests we discussed in lecture.

(*a.*) $\displaystyle\sum_{n=0}^{\infty} 3^n 2^{n+3}$.

(*b.*) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+3}$.

(*c.*) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} \sin\Big[\pi(n{+}1/2)\Big]$

(*d.*) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n-1}$.

(*e.*) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n+3} - \frac{1}{n+4}\right)$