<u>REVIEW FOR TEST 2 OF CALCULUS II:</u>

The first and best line of defense is to complete and understand the homework and lecture examples. Most of the test will be like problems you've done before, they may not be the same format but they should require the same skill set.

• the test is likely to divide up about 20% separation of variables, 15% integrating factor method, 15% homogeneous 2nd order ODE, 50% sequence and series .
(Bonus out of class.)

• recall that I **promised** there would be some difficult integration on the second test. I mentioned in class this would likely happen in the context of the separation of variables problem. Generally, you are expected to be able to do all the same things that were expected in test 1. That also holds in calculus III and all other courses that have calculus II as a pre-requisite.

<u>Sequences :</u>

1. Know your basic limits at infinity. If $f(n) = a_n$ then the limits $\lim_{x\to\infty} f(x)$ and $\lim_{n\to\infty} a_n$ work out the same. Either they both converge to the same value or they both diverge the same way. Technically they are different if we think of x as a real-valued variable verses n which is thought of as a natural number-valued variable (such variables are called *discrete*).

2. Know when and how to use L'Hopital's Rule. (mention you are extending n to be a continuous variable since we cannot differentiate with respect to a function of a discrete variable)

3. Know definitions of increasing, decreasing, bounded above and bounded below. Also know the definition of bounded. Be able to apply the bounded monotonic sequence theorem. (I would give you a push in the right direction if such a question were asked, it might be part of a true/false question) . Items 1,2,4 are much more important than this item for the test.

4. Be able to prove a sequence is decreasing or increasing by appropriate arguments. This either means to apply an easy inequality or use differentiation of the corresponding function of a continuous variable.

<u>Series:</u>

1. What is the sequence of partial sums for $\sum a_n$? Here the notation is meant to indicate several possibilities; $\sum_{n=1}^{\infty} a_n$ or perhaps $\sum_{n=0}^{\infty} a_n$. (*We'd like to allow for some freedom at to where the summation begins.*).

2. How is the series $\sum a_n$ defined ?

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n \qquad \text{where } S_n = \sum_{k=1}^{k} a_k$$

3. When does $\sum a_n$ converge ? What do we mean when we say $\sum a_n$ diverges?

4. What are the two general classes of examples we can actually calculate S_n and hence $\sum a_n$ explicitly ? (we will add another class in later material, but as of now we can only calculate the n-th partial sum for two cases)

5. What is a telescoping series? How do we calculate the value to which the series converges (if it converges)?

6. What is a geometric series? How do we calculate the value to which the series converges (if it converges)?

7. What is the n-th term test? What does it tell you? Is it always useful?

8. What is the integral test? What conditions must you verify in order to use the integral test? What does it tell you? Is it always useful?

9. What is the p-series test? What condition must you verify in order to use the p-series test? What does it tell you? Is it always useful?

10. What is the Alternating Series Test (A.S.T.)? What conditions must you verify in order to use the A.S.T.? What does it tell you? Is it always useful?

11. What is the Ratio Test ? What does it tell you? Is it always useful?

12. What is the Direct Comparison Test (DCT)? What conditions must you verify in order to use the DCT? What does it tell you? Is it always useful? *This test is useless without something to compare to, we usually use either the p-series or geometric series however there are also cute examples based on a comparison to some integral test-testable series. The DCT is not hard, but it does require some imagination and a hint of common sense.*

13. What is the Limit Comparison Test (LCT)? What conditions must you verify in order to use the DCT? What does it tell you? Is it always useful?

14. What is the magnitude of the error in S_n compared to the series $S = \sum a_n$? It is defined to be $|E_{S_n}| = |S - S_n|$. There are many results known that help bound the error, but all you need to know is the error for an alternating series. If $b_n > 0$ and

$$S = b_1 - b_2 + b_3 + \cdots = \sum_{n=0}^{\infty} (-1)^n b_n \quad \text{then} \quad \boxed{|E_{S_n}| \leq b_{n+1}}$$

15. What is absolute convergence? What is conditional convergence? Be able to prove that absolute convergence implies convergence for an arbitrary series. Be able to give a counter-example of the converse; what series is convergent but not absolutely convergent?

16. Given an arbitrary series how do you proceed? What if you are not told which test to use? Which one should you try first? How do you know where to start? *(there is not just one answer to this, there are many reasonable strategies, I recommend thinking about the flow-chart on page 291. Others may find the table posted on the website a useful study aid. Remember that you need to be able to determine convergence or divergence without the notes. Force yourself to work some problems cold. Think about where you got stuck in the quizzes and fix your misunderstanding)*

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Differential Equations:

1. Separate and integrate. (when you can)

2. Know the integrating factor method. Be able to apply the method to solve appropriate first order differential equations.

3. Know the "Recipe". Be able to find general solution to $ay'' + by' + cy = 0$ for any $a, b, c \in \mathbb{R}$.

4. Be able to apply initial conditions to the general solution if appropriate.

5. Be aware of the distinction between an *implicit* and *explicit* solution. Don't waste time solving for y if you don't need to. (sometimes you can't find a formula for the explicit solution)

6. Be ready to think about the meaning of the differential equation as it applies to the solutions. Sometimes we can say things without even solving the solution. For example, $dy/dx = y^2$ indicates it has solutions which are always increasing, or are identically zero because the change in y is clearly either positive or zero. I might ask you to sketch solutions for a simple differential equation w/o solving it analytically. Direction fields are linked to this idea as well, however I would not ask you to sketch a direction field for anything interesting since it demands time you'll need to think about other things. Mathematica is the best way to sketch such direction fields.