REVIEW FOR TEST 3 OF CALCULUS II:

The first and best line of defense is to complete and understand the homework and lecture examples. Most of the test will be like problems you've done before, they may not be the same format but they should require the same skill set.

• the test is likely to divide up about 30% I.O.C and R.O.C, 15% Taylor Series, 15% Binomial Series, 30% geometric series, 10% power series solutions of integrals (Bonus out of class.)

• I wouldn't be surprised if I put questions 5 -8 from Test 2 on this test again.

Series: (Test 3 builds on Test 2, you still need to know all that material, however I narrow the focus a bit here and tell you what is most important)

1. What is the sequence of partial sums for $\sum a_n$? Here the notation is meant to indicate several possibilities; $\sum_{n=1}^{\infty} a_n$ or perhaps $\sum_{n=0}^{\infty} a_n$. (*We'd like to allow for some freedom at to where the summation begins.*).

2. How is the series $\sum a_n$ defined ?

$$\boxed{\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n \qquad \text{where } S_n = \sum_{k=1}^{k} a_k}$$

3. When does $\sum a_n$ converge ? What do we mean when we say $\sum a_n$ diverges?

4. What is a geometric series? How do we calculate the value to which the series converges (if it converges)?

5. What is the n-th term test? What does it tell you? Is it always useful?

6. What is the p-series test? What condition must you verify in order to use the p-series test? What does it tell you? Is it always useful?

7. What is the Alternating Series Test (A.S.T.)? What conditions must you verify in order to use the A.S.T.? What does it tell you? Is it always useful?

8. What is the Ratio Test ? What does it tell you? Is it always useful?

9. What is the magnitude of the error in S_n compared to the series $S = \sum a_n$? It is defined to be $|E_{S_n}| = |S - S_n|$. There are many results known that help bound the error, but all you need to know is the error for an alternating series. If $b_n > 0$ and

$$S = b_1 - b_2 + b_3 + \cdots = \sum_{n=0}^{\infty} (-1)^n b_n \quad \text{then} \quad \boxed{|E_{S_n}| \leq b_{n+1}}$$

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Domain for a Power Series:

1. Know the possibilities for the IOC and ROC.

2. Know what the "open interval of convergence" is relative to the I.O.C.

3. Be able to use the Ratio Test to find the open IOC.

4. Be able to check endpoints using various tests from Test 2. In particular, expect p-series , alternating series and/or n-th term testable series to arise at the endpoints.

5. Be able to identify a given series as geometric. Do you need to check the Ratio Test in this case ?

Geometric Series and Power Series:

1. Be able to find complete power series expansions for functions which match the form $\frac{a}{1-r}$. You may need to do some arithmetic in order for the function to match the formula for the geometric series, as with $\frac{x^3}{2+x}$ for example.

2. Be able to find power series for things like $f(x) = \ln(1 + x^3)$ or $f(x) = \tan^{-1}(x)$ by using the: differentiate, find power series then integrate trick.

3. Be able to find power series for things like $f(x) = \frac{1}{(1+x)^2}$ by using the: integrate, find power series then differentiate to get back to $f(x)$ trick.

4. Look at the supplemental materials for additional examples, also I have a flow chart to illustrate the geometric series trick idea.

5. Be able to take examples above and twist them a bit, like say $f(x) = x^3 \ln(1 + x^3)$ or $f(x) = x^{11} \tan^{-1}(x)$.

6. All the problems listed above you should be able to find the complete series (from $n = 0$ all the way to $n = \infty$). It is of course possible for me to just ask for just the first few terms. The problem statements on the test will clearly state how much of the power series I expect you to list.

Known (Maclaurin) Power Series and Substitutions:

1. You should memorize the power series expansions about zero for sine, cosin, exponential and $f(x) = (1 + x)^k$. You should be able to quickly recite the first three terms in any of those standard examples.

2. Be able to use the known power series expansions from (1) to quickly generate similar power series by some substitution and/or simple algebra. For example, $f(x) = xe^{x2}$ or $f(x) = x + x^2 \sin(\pi x)$ and so forth. These sort of examples you should be able to find complete series expansions for without too much trouble. Of course it is sometimes the case that the first few cases are weird and must be written separately (especially with binomial series examples).

3. Be able to find the first 3 or 4 terms in the power series expansion of $f(x) = \sin(x^2)e^{-2x}$ or $f(x) = \frac{\sin(x)}{(3+x)^{1.5}}$. You should do this by multiplying the known Maclaurin series and keep just enough terms for the requested number of terms. This is probably quicker than a direct application of Taylor's Theorem. By the way, as far as I know there is no "nice" way to do that homework problem $f(x) = \ln(x)/x$ centered around $a = 1$; it's just ugly. You could use the Taylor Series expansion directly, or alternatively use the sneaky equations $\ln(x) = \ln(1 + (x - 1))$ and $\frac{1}{x} = \frac{1}{1+(x-1)}$ to help see how to use geometric series tricks to get power series for $\ln(x)$ and $\frac{1}{x}$, but after that you'd need to multiply the series to get the expansion of $\frac{1}{x}\ln(x)$.

Taylor Series and Integration:

1. What are Taylor Polynomials? Be able to find Taylor polynomials given a particular function and a particular point to center the polynomials. Given a function $f(x)$, what is $T_1(x)$ centered about $a \in dom(f)$?

2. Be able to use Taylor's Theorem to find series expansion of a given function about some given point. For example, find power series representation of $f(x) = \sqrt{x}$ about $a = 1$. I will say explicitly on the test for you to "use Taylor's Theorem" to do the problem. It is true that most any problem on the test could be solved via Taylor's Theorem but that is rarely the best way and often it would take so long you would not be able to finish in time.

3. Know how to find a series solution to an improper integral.

4. Know how to approximate a definite integral using power series techniques. Know the alternating series error estimation Theorem and be able to apply it as we did in the homework at least twice.

   Have a great weekend! Please email me if you need help on some point.

Please See the Handout for further examples, it's posted online and I handed it out Friday morning to those of you who were there. (it should be posted by Saturday morning,)