REVIEW FOR TEST 4 OF CALCULUS II:

The first and best line of defense is to complete and understand the homework and lecture examples.

• the test is likely to divide up about

-25% Parametric Curves,
-20% Polar Coordinates,
-10% Arclength,
-10% Surface Area of Solid of Revolution,
-15% Nonhomogeneous Constant Coefficient DEqns,
-10% averages,
-10% probability density.

• Notice you need a complete and working knowledge of all the trigonometric identities to safely attack many of the problems encountered in these topics. Also, you need to refresh your knowledge of improper integration since for the probability density function question. Finally, the nonhomogeneous DEqn question requires you have a complete and working knowledge of the homogeneous case's "Recipe".

Parametric Curves: important ideas,

1. What is a parametric curve? Is this different than the "parametrization of a curve"? What is required to define a parametric curve in $\mathbb{R}^2$?

2. Know your conic sections. Be able to complete the square to reveal (h, k) for various hyperbolas, and ellipses.

3. Be able to parametrize conic sections in terms of trigonometric or hyperbolic trigonometric functions. Know the graphs of trigonometric or hyperbolic trigonometric functions because they give you all sorts of information about signs, this may help you if I ask to parametrize just part of a particular conic section.

4. A few sample problems, (assume all the curves are given a counterclockwise orientation, this means we want the parametrizations to go in the counterclockwise direction for increasing parameter, which is what we did in nearly all the examples)
   a.) parametrize: $x^2 + y^2 = 7$.
   b.) parametrize: $x^2 + 3y^2 = 3$.
   c.) parametrize: $4x^2 + 9y^2 = 36, \ y \geq 0$.
   d.) parametrize: $x^2 - y^2 = 4, \ x \geq 2$.
   e.) parametrize: $x^2 - y^2 = 4, \ x \leq -2$.
   f.) parametrize: $-4x^2 + y^2 = 9, \ y \geq 3$.
   g.) parametrize: $x^2 - 2x + \frac{1}{4}y^2 = 0, \ x \geq 1, y \geq 0$.
   h.) parametrize: $x^2 - 2x - \frac{1}{4}y^2 = 0, \ x \geq 2$.
   i.) parametrize: $x^2 - 2x - \frac{1}{4}y^2 = 0, \ x \geq 2, y \geq 0$.
   i.) parametrize: $x^2 - 2x + y^2 + 4y = 4$.

5. Be able to graph parametric curves for reasonable examples. Consider using $\frac{dx}{dt}$ and $\frac{dy}{dt}$ to guide your sketch, horizontal and vertical tangents are very helpful in crafting the shape.

6. Given a set of parametric equations for x, y how do you verify that they are in fact a correct parametrization of the graph of a given function or equation?

7. Be able to compute the velocity, acceleration and speed given $\phi(t) = (x(t), y(t))$ represents the position of some particular material object.

8. Find parametric equations for a line segment between two given points.

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Polar Coordinates: essential points

1. Know your trigonometric functions well. You need to be able to graph sine and cosine quickly to glean information to make graphs. Also, transformations of graphs seem to play an important role in some examples, know how to find the period of a function.

2. I will allow (and encourage) the identification of negative r with a reflection of π-radians. We did this throughout the homework in many graphs. I was worried about it to begin with because I know of a pickier version of polar coordinates where $r = \sqrt{x^2 + y^2} \geq 0$ is true without qualification. You are free to use the negative radius reflection rule and also the $\theta \cong \theta + 2k\pi$ for any $k \in \mathbb{Z}$ rule. These identifications are geometrically convenient, but they come at the price of ambiguity for the definition of (r, θ) for a given point.

3. Be able to recognize basic polar graphing examples. For example, $r = 3$ or $\theta = \pi$ or $r = 3\sin(\theta)$.

4. Be able to convert back and forth from polar to Cartesian notation. This interplay can be useful in both directions. Sometimes the polar version is convenient, other times the Cartesian formulation is more familiar.

5. Given a particular point be able to convert from Cartesian to Polar coordinates and vice-versa.

6. Certain homeworks were only reasonable because we had many hours or Mathematics to unravel them. On the test it is more likely I'll test the ones that allowed some nice simple algebraic understanding of. Of course, you can still try the table of values method to gain some insight. I would advocate a multifaceted assault on the problem of graphing polar equations.

Arclength:

1. Be able to set-up arclength integrals for the various cases we have discussed. In particular, we have used x, y or t as independent variables which suggested we integrate the following items with respect to matching bounds:

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}dx, \quad ds = \sqrt{\left(\frac{dx}{dy}\right)^2 + 1}dy, \quad ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}dt$$

2. Beware the homework problems. Those have all the tricks I know of for these problems. Be able to do those calculations.

Surface Area:

1. Be able to set-up surface area integrals for the various cases we have discussed. In particular, recall that $ds = 2\pi r ds$. Here ds can be described in a number of ways depending of which choice of x, y or t as independent variables. Also we saw that $r = y$ for a rotation around the x-axis while $r = x$ for a volume formed by rotation around y-axis.

2. Beware the homework problems. Those have all the tricks I know of for these problems. Be able to do those calculations.

Nonhomogeneous Constant Coefficient ODEqns (undetermined coefficients):

1. Be able to find the homogeneous part of the solution using the "Recipe". When we solve the differential equation $ay'' + by' + cy = g$ you apply the "Recipe" to $ay'' + by' + cy = 0$ this gives us the homogenous part of the solution y_h

2. Be able to form the particular solution on the basis of the type of g that appears in $ay'' + by' + cy = g$. Remember that we formed the particular solution y_p from a linear combination of all the various types of functions that appear in the list $g, g', g'', g''', \ldots$.

3. I do not expect you to understand the "overlap case", the problem on the test will not involve "overlap". Overlap is the mathematics of resonance. An important and sometimes destructive physical phenomenon, sometimes is sought after for amplification in electronics… anyhow I'm leaving that for Math 334 (this semester at least).

4. I have posted additional notes and examples in course content, see page 190f for a list of particular solution guesses. The "naïve" guess will work on the test since I do not test you on overlap. It is also on my webpage at:

   http://www.supermath.info/ma241_nonhomogeneous_188_190k.pdf

5. If I give initial conditions you must apply them to the general solution. The general solution will have two arbitrary constants, however the $A, B, C, \ldots$ (the undetermined coefficients) should be take on specific numerical values in the general solution. The should be determined by you doing algebra after you substitute y_p into the given differential equation.

Averages:

1. Know the definition of the f_{avg} with respect to a closed interval $a \leq x \leq b$;

$$\boxed{f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx}$$

Probability:

1. Know the definition of probability density function. The question on the test will have the form "what value of c must you choose in order that $f(x)$ is a probability distribution".

2. I do not expect you know other things about probability. We have a whole other course for that.

Have a great weekend! Please email me if you need help on some point.