Math 132, Test 3, November 10, 2009 Name: ______________________________.

No graphing calculators or electronic communication of any kind. If you need extra paper please ask. Credit will be awarded for correct content and clarity of presentation. Prepare for math battle. This test has 1500 points. There is a take-home bonus problem at the end with instructions. Make sure to at least attempt each part except the bonus. If I ask for the "complete power series representation" for some function that means I want you to give your answer in the general format

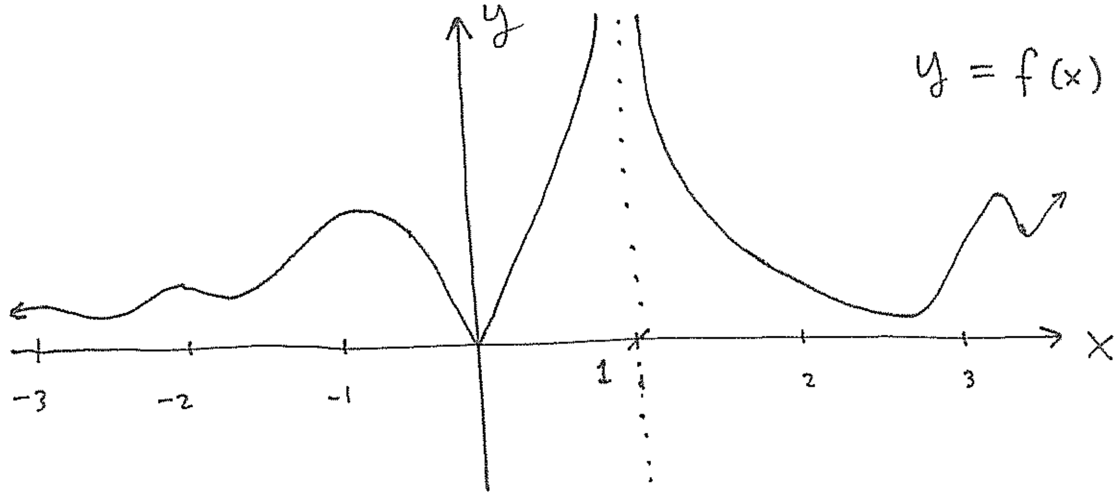
$$\sum_{n=0}^{\infty} c_n (x-a)^n$$

where the coefficients c_n are simplified as much as is possible. Of course in the event that the even and odd terms are different you might write two summations and/or write a few low order terms separately if they break from some pattern. Finally, please assume the all functions are analytic on the interior or their respective domains. I do not expect you to prove that any function on this test has a power series which converges to the function on some domain.

1.[150pts.] Find the IOC and ROC for the power series given below: (justify your answer with appropriate tests as was done in lecture, if you use a particular test then name it.)

$$g(x) = \sum_{n=1}^{\infty} n^2 (2x-3)^n$$

2.[100pts] Given the graph of $y = f(x)$ below answer the questions below the graph.



(YES, THE GRAPH HAS A SHARP CORNER AT (0, 0))

(a.) what appears to be the largest IOC for a power series representation of the function centered at $a = 2$?

(b.) what appears to be the largest IOC for a power series representation of the function centered at $a = -1$?

(c.) sketch $y = T_1(x)$ centered at $a = 2$ on the given graph.

(d.) sketch $y = T_2(x)$ centered at $a = -1$ on the given graph.

3. [150pts] Find the complete power series representation centered about zero for the function given below, state the IOC and ROC for the power series.

$$f(x) = \frac{3}{1+x^2}$$

4.[150pts] Find the complete power series centered at $a = 0$ for the function $f(x) = 3 + \ln(x^4 + 1)$.

5.[100pts] Find the complete power series solution for the following indefinite integral

$$\int x^{13} \cos(x+3)\, dx$$

6.[500pts] these problems ask you to just find a few of the terms (not the complete power series)

(a.) $f(x) = \frac{1}{\sqrt{1-4x}}$ find the first three non-zero terms power series representation centered at $a = 0$.

(b.) find the first three non-zero terms power series centered at $a = 0$ for $f(x) = \tan(x+3)$

(c.) find the first three non-zero terms power series centered at $a = 0$ for

$$f(x) = \sin(x)e^{x} + \frac{1}{1-x}$$

(d.) find the first four non-zero terms power series centered at $a = 1$ for

$f(x) = 2x^3 + 4x^2 - x + 1.$

7.[100pts] The binomial series applied to the function $\sqrt{1+x}$ yields:

$$\sqrt{1+x} = 1 + \tfrac{1}{2}x - \tfrac{1}{8}x^2 + \tfrac{1}{16}x^3 + \cdots.$$

for $x \in (-1, 1)$. Place an upper bound on the error in the approximation

$$\sqrt{1+\tfrac{1}{2}} = 1 + \tfrac{1}{4} - \tfrac{1}{32} + error$$

8.[100pts] The Fourier coefficients for a function $f(x)$ with period $2L = 2\pi$ are

$$a_0 = 1, \quad a_n = 0, \quad b_n = \frac{1}{n}$$

for $n = 1, 2, 3, \ldots$ Use the given coefficients to write the Fourier series for $f(x)$ and then sketch the graph of the terms up to n=1 over the interval $-3\pi \leq x \leq 3\pi$. Is $f(x)$ an even function, and odd function, or neither even nor odd?

9.[150pts] Find the complete power series centered at $a = 0$ for the function $f(x) = \frac{x}{3+x} + xe^{2x}$. Also, calculate $f^{(99)}(0)$. You can leave the value of the derivative in terms of powers and factorials of particular integers, I don't want or expect a decimal answer.

TAKE-HOME BONUS PROBLEM(s):

(return to my office box at or before 8:50am November 11.)

1. [25pts] Calculate the Taylor series for $e^{i\theta}$ where θ is a real variable and $i^2 = -1$.

2. [25pts] Ignore the fact we already have a proof of $e^{i\theta} = \cos(\theta) + i\sin(\theta)$. Rederive Euler's identity $e^{i\theta} = \cos(\theta) + i\sin(\theta)$ from the series found in 1.

3. [50pts] Divide the series given below via long division:

$$f(x) = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad g(y) = \sum_{k=0}^{\infty} \frac{y^k}{k!}$$

Simplify the answer by identifying the resulting series for $f(x)/g(y)$ as a power series in $(x - y)$. Does this make sense?