Same rules as Mission 1. Enjoy,

Problem 91 Show the cardinality of $[0,1)$ and $(2,3)$ are equal in two ways:

(a.) by constructing an explicit bijection from $[0,1)$ to $(2,3)$

(b.) Using the CSB theorem

Problem 92 Find an explicit bijection from $[0,1)$ to $S_1 = \{z \in \mathbb{C} \mid |z| = 1\}$.

Problem 93 Let A and B be sets. Prove that if A is finite then $A \cap B$ is finite.

Problem 94 Assume the continuum hypothesis is true. Order the cardinalities of the following sets in order from smallest to largest.

$$\mathbb{Q},\ \mathbb{N},\ \mathcal{P}(\mathcal{P}(\mathbb{Z})),\ 3\mathbb{Z}+1,\ \mathcal{P}(2\mathbb{N}),\ \mathcal{P}([0,1]),\ \sin^{-1}\{0\},$$

$$\cosh^{-1}\{0\},\ \cosh^{-1}\{2\},\ \mathbb{N}^n, \mathbb{C}, \mathcal{P}(\mathcal{P}\{1,2\}), \mathbb{N}_{47}$$

and S where $S = \{(x,y) \in \mathbb{Q}^2 |\ x^2 + y^2\}$

Problem 95 Prove $\lim\limits_{x\to 1}(3x^3 + 20x^2 + 10x + 14) = 47$ via the $\varepsilon\delta$ definition of the limit.

Problem 96 Prove $\lim\limits_{x\to a}\left(\dfrac{1}{9+x^2}\right) = \dfrac{1}{9+a^2}$ via the $\varepsilon\delta$ definition of the limit.

Problem 97 Consider $f(x) = \frac{1}{x^2+6x+9} - \frac{1}{x^2}$. Show $f(x) = O\left(\frac{1}{x^3}\right)$ as $x \to \infty$.

Problem 98 Consider the sequence $a_n = \frac{n}{1+n}$ for $n \in \mathbb{N}$. Prove $a_n \to 1$ as $n \to \infty$ from the definition of sequential limit.

Problem 99 Let $a_n = \ln(n+2)$. Prove $a_n \to \infty$ as $n \to \infty$ definition of sequential limit..

Problem 100 An open subset of $\mathbb{R}$ is defined to be $U \subseteq \mathbb{R}$ for which every point in U is an **interior point**. In particular, $p \in U$ is interior iff there exists $\varepsilon > 0$ for which $B_\varepsilon(p) \subseteq U$ where $B_\varepsilon(p) = \{x \in \mathbb{R} \mid |x-p| < \varepsilon\}$. Suppose $A, B \subseteq \mathbb{R}$ are nonempty sets. Further, suppose $f : A \to B$ is continuous. Prove that the inverse image under f of an open set is an open set. That is, given open set $V \subseteq \mathbb{R}$, prove $f^{-1}(V)$ is an open set.