Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. No fuzzies thanks. Box your answers where appropriate. Please do not fold. Thanks!

Problem 11 Rewrite the statements below in reasonable-sounding English: here x, y, a, b denote real variables.

(a.) $\forall x \ \ [x \geq 0 \Rightarrow \exists y(y^2 = x)]$

(b.) $\forall x\, [x \leq 0 \Rightarrow \sim \exists y(y = \ln x)]$

(c.) $\exists x \ \forall y(xy = y)$

(d.) $\forall a, b\, [a \neq 0 \Rightarrow \exists x(ax + b = 0)]$

Problem 12 Simplify each of the following statements by moving negation signs inward as much as possible. For example $[\sim (\sim P \vee \exists x\ Q)] \Leftrightarrow [(\sim\sim P) \wedge (\sim (\exists x\ Q))] \Leftrightarrow [P \wedge (\forall x \sim Q)]$.

(a.) $\sim \forall x, y\ \exists z(P \vee \sim \forall u\ Q)$

(b.) $\sim \forall x \sim \exists y \sim \forall z(P \wedge \sim Q)$

Problem 13 Write each sentence below using the restricted quantifier symbolic notation:

(a.) Every number in the set A has a positive square root.

(b.) Given any real number, there are integers bigger than it and integers smaller than it.

(c.) No positive number equals any negative number.

Problem 14 Suppose P is a proposition. Explain why the following proposition is true.

$$(\exists x \in \mathbb{R})(x^2 + 4x + 5 = 0) \Rightarrow P$$

Problem 15 The statment below is false:

$$\forall(a, b, c \in \mathbb{R})(\exists! y_o \in \mathbb{R})(\text{the graph } y = ax^2 + bx + c \text{ has minimum at } y = y_o.)$$

Modify the statement and prove your modified statement is true using calculus.

Problem 16 Prove the sum of even integers is even.

Problem 17 Prove there does not exist an integer which is both even and odd.

Problem 18 Prove the following:

(a.) $(\exists m, n \in \mathbb{Z})(47m + 3n = 1)$

(b.) $(\exists m, n \in \mathbb{Z})(4m + 8n = 1) \Rightarrow (\forall x \in \mathbb{R})(x^2 = 1)$

Problem 19 Suppose x is an odd integer. Prove $x^2 - 9$ is the product of even integers.

Problem 20 Prove $\sqrt{3}$ is not a rational number.