Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. No fuzzies thanks. Box your answers where appropriate. Please do not fold. Thanks!

Problem 21 Let $a, b, c \in \mathbb{R}$ with $a \neq 0$. Prove $ax^2 + bx + c = 0$ has a unique solution iff $b^2 - 4ac = 0$. Your proof should not use the quadratic formula. Rather, your proof is essentially the derivation of the quadratic formula.

Problem 22 Write the set $\{x \in \mathbb{R} \mid x^5 - 16x > 0\}$ as the union of intervals of real numbers. Be sure to justify your answer using an appropriate sign-chart and theory from calculus.

Problem 23 Write the set $\{x \in \mathbb{R} \mid \frac{x^2-3x^2-1}{x^2+8x+30} \leq 0\}$ as the union of intervals of real numbers. Be sure to justify your answer using an appropriate an sign-chart and theory from calculus.

Problem 24 Prove $a \in \mathbb{Z}$ is odd iff there exists $j \in \mathbb{Z}$ for which $a = 2j + 3$.

Problem 25 Prove the sum of three consecutive integers is divisible by 3.

Problem 26 Prove: if x^2 is an even integer then x is an even integer.

Problem 27 Prove: there does not exist a smallest, positive rational number.

Problem 28 Prove: the sum of a rational and irrational number is irrational.

Problem 29 Prove: There exist two positive irrational numbers a, b such that a^b is rational.

Problem 30 Let A, B be sets. Prove: $A \subseteq B$ iff $A \cap B = A$.