Math 200 Mission 4: set theory

Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. No fuzzies thanks. Box your answers where appropriate. Please do not fold. Thanks!

Let A, B, C be sets for this assignment.

Problem 31 Prove: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Problem 32 Prove: If $A \subseteq B$ then $A \cap C \subseteq B \cap C$.

Problem 33 Prove: $A \subseteq B$ iff $\tilde{B} \subseteq \tilde{A}$.

Problem 34 Prove $\bigcup_{n=1}^{\infty} A_n = [0, \infty)$ given $A_n = [0, n)$ for $n \in \mathbb{N}$.

Problem 35 Prove $\bigcap_{n=1}^{\infty} B_n = \{0\}$ given $B_n = (-1/n, 1/n)$ for $n \in \mathbb{N}$.

Problem 36 Prove $A \times (B - C) = (A \times B) - (A \times C)$.

Problem 37 Prove $A - (B \cup C) = (A - B) \cap (A - C)$.

Problem 38 Let $X = \{\alpha, \beta\}$. Find $\mathcal{P}(X)$ and $\mathcal{P}(\mathcal{P}(X))$.

Problem 39 Suppose $\delta > 0$ and $a \in \mathbb{R}$. We define $B_{\delta}(a) = \{x \in \mathbb{R} \mid |x - a| < \delta\}$. Suppose $B_{\delta}(a) \cap (0,1) \neq \emptyset$. Prove for each $x_o \in B_{\delta}(a) \cap (0,1)$ there exists $\varepsilon > 0$ for which $B_{\varepsilon}(x_o) \subseteq B_{\delta}(a) \cap (0,1)$.

Problem 40 Suppose \mathcal{T} is a topology for X. We define $U \subseteq X$ to be a closed set iff X - U is an open set. Suppose U and V are closed sets in X. Show $U \cap V$ is a closed set.