Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. No fuzzies thanks. Box your answers where appropriate. Please do not fold. Thanks!

Problem 41 Prove $\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$ for each $n \in \mathbb{N}$.

Problem 42 Prove $4n < 2^n$ for all integers $n \geq 5$.

Problem 43 Prove the sum of the first n-odd numbers is n^2.

Problem 44 Every power of 13 can be written as a sum of two squares.

Problem 45 Prove the sum of the interior angles of an n-sided convex polygon[1] is $(n-2)\pi$ for all $n \in \mathbb{N}$ with $n \geq 3$.

Problem 46 Every positive integer n can be written as a sum of distinct nonnegative integer powers of two.

Problem 47 Some related induction proofs,

(a.) Prove $\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$.

(b.) Suppose a sequence of real numbers a_k satisfies $\sum_{k=1}^{n} a_k^3 = \left(\sum_{k=1}^{n} a_k\right)^2$. Prove, by induction on n, that necessarily $a_k = k$ for all $k \in \mathbb{N}$.

Problem 48 Let $A_i, B_{ij} \in \mathbb{R}$ for $i, j \in \mathbb{N}$. Prove $\sum_{i=1}^{n}\sum_{j=1}^{m} A_i B_{ij} = \sum_{i=1}^{n} A_i \sum_{j=1}^{m} B_{ij}$.

Problem 49 Suppose f and g are smooth functions on $\mathbb{R}$. Prove for any $n \in \mathbb{N}$,

$$(fg)^{(n)} = \sum_{k=0}^{n} \binom{n}{k} f^{(n-k)} g^{(k)}.$$

Problem 50 Let $\prod_{i=1}^{1} a_i = a_1$ and recursively define $\prod_{i=1}^{n+1} a_i = a_{n+1} \prod_{i=1}^{n} a_i$ for all $n \in \mathbb{N}$. Let f_i be a positive differentiable function. Prove, for $n \in \mathbb{N}$,

$$\frac{d}{dx}\prod_{i=1}^{n} f_i = \sum_{k=1}^{n} \frac{df_k}{dx}\frac{1}{f_k}\prod_{i=1}^{n} f_i.$$

[1] a convex polygon is one for which any line-segment connecting two points on the polygon is contained within the polygon