Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. No fuzzies thanks. Box your answers where appropriate. Please do not fold. Thanks!

Problem 51 We defined the imaginary exponential as $e^{i\theta} = \cos\theta + i\sin\theta$ for $\theta \in \mathbb{R}$. Prove that $e^{i\theta}e^{i\beta} = e^{i(\theta+\beta)}$ is true for all $\theta, \beta \in \mathbb{R}$. You will need to cite two results from trigonometry. Continuing, show $(e^{i\theta})^n = e^{in\theta}$ for all $n \in \mathbb{N}$. Also, what identities does this identity yield in $n = 2$ case and $n = 6$ cases ? (use binomial theorem to expand)

Problem 52 For any $x \in \mathbb{R}$ and $n \in \mathbb{N}$, $|\sin(nx)| \leq n|\sin(x)|$.

Problem 53 Let $R = \{(1,2),(1,3),(2,3),(2,4)\}$ and $S = \{(4,3),(4,2),(3,2),(3,1)\}$.

(a.) Find $R \circ S$ and $S \circ R$

(b.) Find R^{-1} and S^{-1}.

(c.) Verify $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$.

Problem 54 Graph each relation below and find $Dom(R)$ and $Rng(R)$.

(a.) $R = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1 \text{ or } x^2 + 4x + y^2 = 0\}$

(b.) $R = \{(x,y) \in \mathbb{Z}^2 \mid x^2 = y\}$

(c.) $R = \{(x,n) \in \mathbb{R} \times \mathbb{N} \mid \cos(x) = n\}$

Problem 55 Let $x, y \in \mathbb{R}$ and define $x \sim y$ iff $xy > 0$ or $x = y = 0$. If $\sim$ is an equivalence relation then find the partition of $\mathbb{R}$ to which it corresponds. Otherwise, show $\sim$ is not an equivalence relation on $\mathbb{R}$.

Problem 56 Let A be a set and suppose R is an equivalence relation on A. Prove the following:
If $x, y \in A$ then $\sim (xRy)$ iff $[x]$ and $[y]$ are disjoint.

Problem 57 Let $S = \{f : \mathbb{R} \to \mathbb{R} \mid (\forall x)(f''(x) \in \mathbb{R})\}$. Define $f \sim g$ iff $f''(x) = g''(x)$ for all $x \in \mathbb{R}$. Prove $\sim$ is an equivalence relation on S. Find every possible representative of the equivalence class $[0]$. Characterize the equivalence classes of $\sim$.

Problem 58 Find all equivalence relations on the set $\{1,2,3,4\}$.

Problem 59 Let $S = \{(a,b) \in \mathbb{Z}^2 \mid b \neq 0\}$. Let $(a,b),(c,d) \in S$ and define $(a,b) \sim (c,d)$ iff $ad - bc = 0$.

(a.) show $\sim$ is an equivalence relation on S,

(b.) show $[(ka,kb)] = [(a,b)]$ for all $k \in \mathbb{N}$.

Problem 60 Let $x, y \in \mathbb{R}$ then define $x \sim y$ iff $y - x = 2\pi k$ for some $k \in \mathbb{Z}$. Prove $\sim$ is an equivalence relation.