Same instructions as before.

Problem 61 Let R_1 and R_2 be equivalence relations on a set A.

(a.) Show $R_1 \cap R_2$ is an equivalence relation.

(b.) Show $R_1 \cup R_2$ need not be an equivalence relation.

Problem 62 Let X be a set and suppose $\mathcal{P}(X)$ is the power set of X. Let $U, V \in \mathcal{P}(X)$ then $U\,R\,V$ iff $U \subseteq V$ and $U \neq V$. Prove R defines an irreflexive partial ordering. Also, explain why R is not an irreflexive total ordering.

Problem 63 Use the Euclidean algorithm to find $a, b \in \mathbb{Z}$ for which $ax + by = gcd(x, y)$ given that $x = 517$ and $y = 141$.

Problem 64 Consider $\mathbb{Z}_{323}$. Find the multiplicative inverse of $[x]$ in $\mathbb{Z}_{323}$ if possible, if not show that there exists $[y] \in \mathbb{Z}_{323}$ for which $[z][y] = [0]$ (this means $[x]$ is a **zero divisor**). Please use the Euclidean algorithm to aid your analysis where appropriate.

(a.) $x = 322$

(b.) $x = 104$

(c.) $x = 10$

Problem 65 Simplify $[123456789]$ in $\mathbb{Z}_9$. (no calculator allowed)

Problem 66 Which digits must we substitute for a and b in $30a0b03$ so that the resulting integer is divisible by 13 ?

Problem 67 Prove $2^n + 6 \cdot 9^n$ is always divisible by 7 for any $n \in \mathbb{N}$.

Problem 68 Suppose $m \neq 0$ and $a, b \in \mathbb{Z}$. Prove $ma \mid mb$ iff $a \mid b$.

Problem 69 Show $15 \mid (4^{2n+1} - 7^{4n-2})$ for all $n \in \mathbb{N}$. *Hint:* $15 = 3\dot{5}$.

Problem 70 Find the last digits of the following numbers:

(a.) 4^{100}

(b.) 2006^3

(c.) 923^{2006}

(d.) 7^{728}

(e.) 9^{1234}