Same instructions as before.

Problem 71 For how many positive integral values of $x \leq 100$ is $3^x - x^2$ divisible by 5 ?

Problem 72 Let $\mathbb{Z}_{10} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ with arithmetic modulo 10. If $x \in \mathbb{Z}_n$ then we define the **additive order** of x to be the smallest k for which $k \cdot x = 0$ where $k \cdot x = \sum_{i=1}^{k} x$. The **multiplicative order** of x is the smallest $k \in \mathbb{Z}_{\geq 0}$ for which $x^k = 1$ if such k exists.

(a.) calculate the additive order of each number in $\mathbb{Z}_{10}$

(b.) for appropriate numbers, calculate the multiplicative order of each number in $\mathbb{Z}_{10}$

(c.) explain the relation between the previous parts, which numbers have multiplicative orders which are defined, which do not ?

Problem 73 Find the domain of each relation given below and determine if f is a function.

(a.) Let $f = \{(x, y) \in \mathbb{N}^2 \mid y^2 = x\}$

(b.) Let $f = \{(x, y) \in \mathbb{Z}^2 \mid y^2 = x\}$

Problem 74 Suppose x is a real variable. Let $f(x) = \ln(x^2 + 8x + 10)$

(a.) find $dom(f)$.

(b.) prove or disprove f is one-to-one

Problem 75 Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x, y) = (x + y, 2x - y)$ for $(x, y) \in \mathbb{R}^2$

(a.) prove or disprove f is onto

(b.) prove or disprove f is one-to-one

Problem 76 Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \dfrac{x}{x-3}$ for $x \neq 3$ and $f(3) = 1$.

(a.) prove or disprove f is onto

(b.) prove or disprove f is one-to-one

Problem 77 Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3 + 10^{3x^2-1}$ for $x \in \mathbb{R}$.

(a.) prove or disprove f is onto

(b.) prove or disprove f is one-to-one

Problem 78 Let $f : \mathbb{Z}_n \to \mathbb{Z}_m$ be defined by $f([x]_n) = 3[x + 1]_m$.

(a.) let $n = 10$ and $m = 5$, prove or disprove f is a function

(b.) let $n = 4$ and $m = 8$, prove or disprove f is a function

Problem 79 Let A, B, C be nonempty sets. Prove: If $f : A \to B$ and $g : B \to C$ are functions such that $g \circ f$ is injective then f is injective.

Problem 80 Let A, B_1, B_2 be nonempty sets. Suppose $f_1 : A \to B_1$ and $f_2 : A \to B_2$ are functions. We define $f = f_1 \times f_2 : A \to B \times C$ by $f(x) = (f_1(x), f_2(x))$ for all $x \in A$.

(a.) Prove or disprove: $f = f_1 \times f_2$ is a function

(b.) Prove or disprove: if f_1 or f_2 is injective then $f_1 \times f_2$ is injective

(c.) Prove or disprove: if f_1 and f_2 is onto then $f_1 \times f_2$ is onto.