Same instructions as in previous missions. Thanks!

Reminder: Calculus I is a prerequisite for this course. I do expect you know Calculus I and are able to apply it to prove assertions about functions. In particular, graphs of functions can be understood by analysis of the derivative of a function and continuity and the IVT are important to the formulation of arguments. Beyond this, I also assume you have a complete and working knowledge of trigonometry and highschool algebra.

Problem 81 Let $f(x) = |x^2 - 9|$ find the following.

(a.) $f([0,1])$ **(b.)** $f^{-1}([-1,1])$ **(c.)** Let $c \in \mathbb{R}$, find $f^{-1}(\{c\})$.

Problem 82 Let $f(x) = x^2 - 6x + 11$. Calculate the following in interval notation:

(a.) $f([0,4])$ **(b.)** $f^{-1}((2,3])$ **(c.)** $f^{-1}([0,1])$.

Problem 83 Suppose $f(x) = \sqrt[3]{28x+8}$ defines $f : \mathbb{R} \to \mathbb{R}$. Prove f is a bijection.

Problem 84 When we consider $f(x) = \cos(x)$ then the standard local inverse for is given by $\cos^{-1} : [0,\pi] \to [-1,1]$. This is just a choice, there are infinitely many others.

(a.) Let $k \in \mathbb{Z}$, find a local inverse for f on $[2k\pi, (2k+1)\pi]$.

(b.) Let $k \in \mathbb{Z}$, find a local inverse for f on $[(2k+1)\pi, 2k\pi]$.

Problem 85 Let $f(x) = 47 + (x-1)(x-2)\cdots(x-n)$ for some $n \in \mathbb{N}$. Find $f^{-1}\{47\}$.

Problem 86 Let $f(x) = ax^3 + bx^2 + cx + d$ where $a,b,c,d \in \mathbb{R}$. What condition on the coefficients a,b,c,d is necessary if f^{-1} exists. Given this condition, find $f^{-1}([d-c+b-a, a+b+c+d])$.

Problem 87 Let f be a function and $C \subseteq dom(f)$. Show $C \subseteq f^{-1}(f(C))$. Also, give an example which demonstrates $\subseteq$ cannot generally be replaced with $=$.

Problem 88 Let A, B be nonempty sets with $S, T \subseteq A$. Let $f : A \to B$ be a function, prove that:

(a.) $f(S \cup T) = f(S) \cup f(T)$

(b.) $f(S \cap T) \subseteq f(S) \cap f(T)$

(c.) if f is injective then $f(S \cap T) = f(S) \cap f(T)$

Problem 89 If $f : S \to T$ and $A, B \subseteq T$ then $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.

Problem 90 Let $F : \mathbb{R}^2 \to \mathbb{R}$ be defined by $F(x,y) = (y - x^2)^2$

(a.) Let $c \in \mathbb{R}$, find $F^{-1}\{c\}$.

(b.) Describe $\mathbb{R}^2/\sim$ where $(x_1,y_1) \sim (x_2,y_2)$ iff $F(x_1,y_1) = F(x_2,y_2)$

(c.) Let $G : \mathbb{R}^2 \to [0,\infty)$ be defined by $G(x,y) = F(x,y)$ for all $(x,y) \in \mathbb{R}^2$. Verify $\bar{G} : \mathbb{R}^2/\sim \to [0,\infty)$ defined by $\bar{G}([(x,y)]) = G(x,y)$ is a bijection.

(d.) If $S_{(a,b)} = \{(at, bt) \mid t \in [0,\infty)\}$ then for which $(a,b) \in \mathbb{R}^2$ is $F|_{S_{(a,b)}}$ one-to-one ?