Name:

MATH 221 BOSS FIGHT 2 BONUS QUIZ (10PTS)

You may use a non-graphing calculator on this test and you are allowed a page of notes front and back. You have 10 minutes. Most of that time is for Problem 2. Be sure to box your answers, thanks!

Problem 1: (5pts) Let $\gamma = \{\frac{1}{\sqrt{3}}(1,1,1), \frac{1}{\sqrt{2}}(-1,0,1)\}$ give a basis for $W = span(\gamma)$. Calculate $[(a,b,c)]_\gamma$ given that $(a,b,c) \in W$.

Since $\gamma = \{V_1, V_2\}$ has $V_1 \cdot V_1 = 1$, $V_2 \cdot V_2 = 2$ and $V_1 \cdot V_2 = 0$ we see γ is orthonormal. Therefore,

$$[(a,b,c)]_\gamma = \begin{bmatrix} (a,b,c) \cdot V_1 \\ (a,b,c) \cdot V_2 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} \frac{a+b+c}{\sqrt{3}} \\ \frac{-a+c}{\sqrt{2}} \end{bmatrix}}$$

Problem 2: (5pts) Let $A = \begin{array}{c} \begin{matrix} V_1 & V_2 & V_3 \end{matrix} \\ \begin{bmatrix} 3 & 6 & 4 \\ 3 & 1 & 4 \\ 3 & 6 & 0 \\ 3 & 1 & 0 \end{bmatrix} \end{array}$. Find the QR-decomposition of the given matrix.

Begin by forming an orthonormal basis for Col (A)

$$V_1' = (1,1,1,1) \quad , \quad \underline{V_1' \cdot V_1' = 4}.$$

$$\tilde{V}_2' = (6,1,6,1) - \left(\frac{V_1' \cdot V_2}{V_1' \cdot V_1'}\right)(1,1,1,1)$$

$$= (6,1,6,1) - (7/2, 7/2, 7/2, 7/2)$$

$$= (5/2, -5/2, 5/2, -5/2)$$

$$= \frac{5}{2}(1,-1,1,-1) \Rightarrow \underbrace{V_2' = (1,-1,1,-1)}_{\text{get rid of pesky fraction.}}, \quad \underline{V_2' \cdot V_2' = 4}.$$

$$\tilde{V}_3' = (4,4,0,0) - \left(\frac{V_3 \cdot V_1'}{V_1' \cdot V_1'}\right)V_1' - \left(\frac{V_3 \cdot V_2'}{V_2' \cdot V_2'}\right)V_2'$$

$$= (4,4,0,0) - 2(1,1,1,1)$$

$$= (2,2,-2,-2) \Rightarrow V_3' = (1,1,-1,-1), \quad \underline{V_3' \cdot V_3' = 4}.$$

Thus q_1, q_2, q_3 are given by normalizing V_1', V_2', V_3' hence,

$$Q = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

$$Q^T A = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} 3 & 6 & 4 \\ 3 & 1 & 4 \\ 3 & 6 & 0 \\ 3 & 1 & 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 12 & 14 & 8 \\ 0 & 10 & 0 \\ 0 & 0 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 7 & 4 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

$$\boxed{A = QR = \begin{bmatrix} 1/2 & 1/2 & 1/2 \\ 1/2 & -1/2 & 1/2 \\ 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & -1/2 \end{bmatrix}\begin{bmatrix} 6 & 7 & 4 \\ 0 & 5 & 0 \\ 0 & 0 & 4 \end{bmatrix}}$$