Name:

MATH 221	BOSS FIGHT 1: MATRICES, LINEAR EQNS, RANK AND NULLITY (200+20PTS)

Show your work where appropriate and box answers. **Standard parametric form** means use non-pivotal variables as parameters for solution set. Thanks! You may use a non-graphing calculator on this test and you are allowed a 3×5 inch card of notes.

Problem 1: (24pts) Find the solution of $\left\{ \begin{array}{c} x + y - z = 7 \\ 2x + y - 2z = 8 \\ 3x - 4y + z = 7 \end{array} \right\}$

Problem 2: (16pts) Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 9 & 9 & 9 \end{bmatrix}$ let e_1, e_2, e_3, e_4 denote the standard basis for $\mathbb{R}^4$ and $\bar{e}_1, \bar{e}_2, \bar{e}_3$ denote the standard basis for $\mathbb{R}^3$. Calculate the following:

(a.) $A[e_1|e_4]$

(b.) $(\, A^T[\bar{e}_1|\bar{e}_2]\,)^T$

Problem 3: (30pt) Let $a, b, c \in \mathbb{R}$. Calculate $\begin{bmatrix} 1 & -2 & 0 \\ 1 & -2 & 1 \\ 1 & -1 & 0 \end{bmatrix}^{-1}$ and solve $\left\{ \begin{array}{rcl} x_1 - 2x_2 & = & a \\ x_1 - 2x_3 + x_3 & = & b \\ x_1 - x_2 & = & c \end{array} \right\}$.

Problem 4: Let $v_1 = (1, 2, 2, 1)$ and $v_2 = (1, 3, 2, 4)$ and $w = (1, 0, 6, 4)$.

(a.) (10pts) Determine if $w \in span\{v_1, v_2\}$

(b.) (10pts) Determine if $\{v_1, v_2, w\}$ is a LI set.

Problem 5: (10pts) Let $A = \begin{bmatrix} 3 & 1 & 0 & 0 \\ 3 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 3 & 3 \end{bmatrix} \oplus \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. Calculate A^{-1}.

Problem 6: (20pts) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 3 \end{bmatrix}$. Calculate:

(a) AA^T

(b) $A^T A$

Problem 7: (20pts) Find the augmented coefficient matrix $[A|b]$ for:

$$\begin{aligned} x_1 + 2x_2 + 2x_3 + 4x_4 &= 0 \\ -2x_1 - 4x_2 - 3x_3 &= -3 \end{aligned}$$

and use row operations to calculate rref$[A|b]$. Write the general solution of $Ax = b$ in standard parametric form.

Problem 8: You are given that:

$$A = \begin{bmatrix} 1 & -3 & 2 & -1 \\ 3 & -9 & 2 & -7 \\ 3 & -9 & 2 & -7 \end{bmatrix} \qquad \text{has} \qquad \text{rref}(A) = \begin{bmatrix} 1 & -3 & 0 & -3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(a.) (5pts) Find the basis for Col(A) via the CCP,

(b.) (10pts) Find the basis for Null(A).

Problem 9: If possible, write M as the product of elementary matrices. If not, explain why it is not possible.

(a.) (5pts) $M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(b.) (10pts) $M = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Problem 10: (10pts) Let W be the set of quadratic polynomials whose graphs contain the point $(3, 0)$. Express W as a span.

Choose your adventure: pick one of the following two problems for credit:

Problem 11: (15pt) Let $A \in \mathbb{R}^{m \times p}$ and $B \in \mathbb{R}^{p \times m}$. Prove $tr(AB) = tr(BA)$.

Problem 12: (15pt) Let $rref[v_1|v_2|v_3|e_3|e_2|e_1] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 7 & 0 \\ 0 & 1 & 0 & 0 & 7 & 4 \\ 0 & 0 & 1 & 3 & 0 & 1 \end{array}\right]$. If $A = [v_1|v_2|v_3]$ then calculate A^{-1} in terms of v_1, v_2, v_3.

Problem 13: (15pts) Suppose $G = \{A \mid A^T J A = J\} \subseteq \mathbb{R}^{n \times n}$ where J is an invertible matrix. Show that if $A, B \in G$ then $AB \in G$ and $A^{-1} \in G$.