Name:

MATH 221 BOSS FIGHT 2: LINEAR MAPS, COORDINATES & ORTHONORMALITY (200+20PTS)

You may use a non-graphing calculator on this test and you are allowed a page of notes front and back.

Problem 1: Let $T(x, y, z, w) = (x + 2y + 3z + 4w, 2x - 3w, 2y + z)$

(a.) (8pts) Calculate the standard matrix $[T]$

(b.) (4pts) Calculate $rref[T]$

(c.) (4pt) Is T an surjective mapping ?

(d.) (4pt) Is T an injective mapping ?

Problem 2: (10pts) Suppose A is a 2×2 matrix with $det(3A) = 900$ and B is a 3×3 matrix with $det(B) = 5$. If $M = \left[\begin{array}{c|c} A & 0 \\ \hline 0 & B^{-1} \end{array}\right]$ then calculate $det(M)$.

Problem 3: (30pts) Let $S = \{(1,0,1,0),(0,3,0,4),(1,-4,1,3)\}$. Find the following:

(a.) (10pts) an orthonormal basis β_1 for $span(S)$,

(b.) (10pts) an orthonormal basis for β_2 for $S^{\perp}$

(c.) (10pts) Let $v = (a,b,c,d)$. Find $v_1 \in span(S)$ and $v_2 \in S^{\perp}$ such that $v = v_1 + v_2$.

Problem 4: (10pt) Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 0 \\ 3 & 2 & 1 & 0 & 0 \\ 2 & 2 & 3 & 4 & 5 \end{bmatrix}$. Calculate $det(A)$.

Problem 5: (10pts) Find the line which is closest to the points $(-2,-6),(-1,1),(0,9),(1,13)$.

Problem 6: (10pts) Let $T(f) = x\frac{df}{dx} + 3xf$ for each $f = a + bx + cx^2$ in $P_2(\mathbb{R})$. This is a linear transformation. Let $\beta = \{1, x, x^2\}$ and $\gamma = \{1, x, x^2, x^3\}$. Calculate $[T]_{\beta,\gamma}$.

Problem 7: (25pts) Let $A = \begin{bmatrix} 3 & 6 \\ 3 & 1 \\ 3 & 6 \\ 3 & 1 \end{bmatrix}$. Find the QR-decomposition of the given matrix.

Problem 8: (15pts) Suppose $W = \text{span}\{(3,3,3,3), (6,1,6,1)\}$. Find the point on W which is closest to (a,b,c,d) and determine how far $(0,6,7,0)$ is from W.

Problem 9: (35pt) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ -3 & 2 & 0 \end{bmatrix}$.

(a.) (10pts) Calculate $det(A)$,

(b.) (10pts) Calculate $det(B)$,

(c.) (15pts) Calculate the inverse of the invertible matrix in this problem.

Problem 10: (15pts) Let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$. Define $\beta = \{v_1, v_2, v_3\}$. Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ has $T(v_1) = v_1$ and $T(v_2) = 2v_2$ and $T(v_3) = 3v_3$. Calculate $[T]_{\beta,\beta}$ and express $[T]$ as a product of three explicit matrices.

Choose your adventure: work either (a.) or (b.) for credit

Problem 11: (10pts) Prove W given below is a subspace by use of an appropriate theorem from lecture:

(a.) $W = \{A \in \mathbb{R}^{n\times n} \mid A^T = A\}$.

(b.) $W = \{a + bx^2 + cx^4 \mid a, b, c \in \mathbb{R}\}$

Problem 12: (10pts) Let $A \in \mathbb{R}^{3\times 3}$. Suppose $T(x) = Ax$ defines $T : \mathbb{R}^3 \to \mathbb{R}^3$. You're given that

$$T(v_1) = (1, 2, 3), \quad T(v_2) = (3, 4, 4), \quad T(v_3) = (4, 6, 7).$$

Suppose $det[v_1|v_2|v_3] = 6$ and $tr[v_1|v_2|v_3] = 3$. Calculate the determinant and trace of A.

Problem 13: (20pts) Suppose $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation for which

$$T\left(\begin{bmatrix} 2/3 \\ 2/3 \\ 1/3 \end{bmatrix}\right) = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} \quad \& \quad T\left(\begin{bmatrix} 2/3 \\ -1/3 \\ -2/3 \end{bmatrix}\right) = \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix} \quad \& \quad T\left(\begin{bmatrix} -1/3 \\ 2/3 \\ -2/3 \end{bmatrix}\right) = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$$

Find the standard matrix of T.