Required Documents and Files:

(I.) Matlab .m file with clearly labeled sections and commented code.

(II.) Report pdf which answers questions posed in this project with a combination of complete sentences and claims which are supported by the Matlab file.

Philosophy of Project: use built-in Matlab commands. No need to reinvent the wheel. I am fairly sure there exist Matlab commands to complete most of the tasks I ask below. I am not here to tell you which commands to use, but I am happy to explain any unclear notation in the problem set-up. I do expect you use Matlab to calculate the bulk of what is asked. To be clear, I will not give much partial credit if you were to foolishly attempt these via direct pen and paper calculation (except where I explicitly say use "pen and paper")

Problem 1: Orthonormalize the following sets of vectors via the Gram-Schmidt algorithm:

(I.) $S = \{(1,2,2,0,1),(1,2,2,7,1),(0,0,2,4,0),(0,0,0,1,0)\}$

(II.) $T_4 = \{e_1 + 2e_2 + \cdots + ie_i \mid 1 \leq i \leq n-1\} \subset \mathbb{R}^n$ for $n = 4$,

(III.) $T_8 = \{e_1 + 2e_2 + \cdots + ie_i \mid 1 \leq i \leq n-1\} \subset \mathbb{R}^n$ for $n = 8$

(IV.) $U_5 = \{\cos(\pi i)e_i + 2^i e_{i+1} \mid 1 \leq i \leq n-1\} \subset \mathbb{R}^n$ for $n = 5$

(V.) $U_{10} = \{\cos(\pi i)e_i + 2^i e_{i+1} \mid 1 \leq i \leq n-1\} \subset \mathbb{R}^n$ for $n = 10$

Remark: if any of these sets are not LI please let me know so I can give additional instructions. Please use the labels **I. II. III. IV. and V.** in what follows as appropriate.

Problem 2: Let $\{u_1, \dots, u_{n-1}\}$ denote the orthonormalized set of vectors found in a particular case of the previous problem. Find a vector $u_n \in \mathbb{R}^n$ which makes $\{u_1, \dots, u_{n-1}, u_n\}$ an orthonormal basis. Do this for each case of the previous problem.

Problem 3: Let $\beta = \{u_1, \dots, u_{n-1}, u_n\}$ be an orthonormal basis for $\mathbb{R}^n$ and define

$$B = 2u_1u_1^T + 2u_2u_2^T + \cdots + 2u_nu_n^T + u_1u_2^T + u_2u_3^T + \cdots + u_{n-1}u_n^T$$

(a.) Show by direct pen and paper calculation that β is a generalized eigenbasis for B. In particular, show that β is a n-chain eigenvalue $\lambda = 2$. In particular, show u_n is a generalized eigenvector of order n with eigenvalue $\lambda = 2$.

(b.) Find generalized eigenvectors for B via Matlab for B constructed for the orthonormal basis you constructed in Problem 2. Hopefully these vectors are precisely the basis you used to construct B.

(c.) For each case, calculate $[\beta]^{-1}B[\beta]$ and comment on the form of the matrix.

(d.) For each case, calculate e^{tB} and verify the identity $det(e^{tB}) = exp(trace(tB))$

(e.) For each case, solve $\frac{dx}{dt} = Bx$ given $x(0) = (1, 2, \dots, n)$.

Problem 4: Use the basis from case **III.** to build a matrix M which has Jordan form

$$J_2(2) \oplus J_4(3) \oplus J_2(6)$$

check that $[\beta]^{-1}M[\beta] = J_2(2) \oplus J_4(3) \oplus J_2(6)$.

Problem 5: Verify the Cayley Hamilton Theorem for each B constructed in Problem 3.