Math 221 Mission 1: linear equations and row reduction technique (30pts)

Show your work and box answers. This pdf should be printed and your solution should be handwritten on the printout. Once complete, please staple in upper left corner. Thanks.

**Suggested Reading** You may find the following helpful resources beyond lecture,

**(a.)** Chapter 1 of my lecture notes for Math 221

**(b.)** §1.1, 1.2 of Lay's *Linear Algebra*

**(c.)** Reduced Row Echelon Web Calculator
(see http://www.math.odu.edu/ bogacki/cgi-bin/lat.cgi?c=rref
please consider using this to check your answers, or better yet, use Matlab, but the answers and work must be shown in handwritten detailed work here. To be clear, you cannot just use Matlab and printout the result. Not yet anyway, all in good time.)

**Problem 1:** Solve the following system by back-substitution. In particular, solve the last equation and use the solution to simplify the preceding equation, rinse and repeat.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 &= 55 \\ 2x_2 + 3x_3 + 4x_4 + 5x_5 &= 54 \\ 3x_3 + 4x_4 + 5x_5 &= 50 \\ 4x_4 + 5x_5 &= 41 \\ 5x_5 &= 25 \end{aligned}$$

**Problem 2:** Solve each system and graph each equation. Explain why the system is inconsistent for the inconsistent systems.

**(a.)** $y = x + 2$ and $y = -x + 3$

**(b.)** $2x + 3y = 1$ and $4x + 6y = 2$

**(c.)** $y = x + 1$ and $y = 1 - x$ and $2x - 2y = 4$

**Problem 3:** Let $a, b$ be constants. Solve $\left\{ \begin{array}{ll} x + 2y & = a \\ 3x + 4y & = b \end{array} \right\}$. Your answer will involve $a$ and $b$.

**Problem 4:** Solve the following system of equations via row-reduction of the augmented coefficient matrix.

$$\left\{\begin{array}{rcl} x+2y & = & 3z-w \\ 2x-y+z & = & w+2 \\ y+z+w & = & 6 \\ 3z+x & = & 12+y-w \end{array}\right\}$$

**Problem 5:** Write down the augmented coefficient matrix $[A|b]$ for each system given below:

**(a.)** $$\left\{ \begin{array}{rcl} x_1 + x_2 + 3x_3 + 4x_4 & = & 12 \\ 2x_2 + 4x_3 - 2x_4 & = & 16 \\ 3x_1 - x_2 + 6x_3 + x_4 & = & 19 \end{array} \right\}$$

**(b.)** $$\left\{ \begin{array}{lcl} 2s + t & = & 1 \\ 3s - t & = & 3 \\ s + 4t & = & 0 \end{array} \right\}$$

**(c.)** $$\left\{ \begin{array}{rcl} 2x_1 + x_2 + x_3 & = & 0 \\ 3x_1 - x_2 + 3x_3 & = & 0 \\ x_1 + 4x_2 & = & 0 \end{array} \right\}$$

**(d.)** $$\left\{ \begin{array}{rcl} x_1 - 2x_2 + 3x_3 - 6x_4 & = & 1 \\ 2x_1 - 7x_2 + x_3 + x_4 & = & 2 \end{array} \right\}$$

**(e.)** $$\left\{ \begin{array}{rcl} 3x_4 + x_6 - x_7 & = & 1 \\ 2x_1 - 2x_2 - 3x_3 - 4x_4 - 5x_5 - 6x_6 - 7x_7 & = & 0 \end{array} \right\}$$

**Problem 6:** Use the row-reduction technique to calculate rref$[A|b]$ for each system given in the previous problem and write down the solution set in standard form for each system. (attach the solutions in order, clearly labeled after this page)

**Problem 7:** Consider the following systems:

$$\textbf{(I.)} \left\{ \begin{array}{rcr} x_1 + x_3 & = & 1 \\ 2x_1 + 3x_2 + 8x_3 & = & 1 \\ 3x_1 - x_2 + x_3 & = & -4 \end{array} \right\}$$

$$\textbf{(II.)} \left\{ \begin{array}{rcr} x_1 + x_3 & = & 1 \\ 2x_1 + 3x_2 + 8x_3 & = & 5 \\ 3x_1 - x_2 + x_3 & = & 2 \end{array} \right\}$$

These systems share the same matrix of coefficients $A$ whereas the inhomogeneous terms differ. If $[A|b_I]$ is the augmented coefficient matrix for **(I.)** and $[A|b_{II}]$ is the augmented coefficient matrix for **(II.)** then we may solve both systems by row-reducing $[A|b_I|b_{II}]$.

**(a.)** write down $[A|b_I|b_{II}]$

**(b.)** use row-reduction to calculate rref$[A|b_I|b_{II}]$

**(c.)** write down the standard solution to system **(I.)** and **(II.)**, or state no solution.

**Problem 8:** Let $a, b, c$ be constants. Consider $\left\{ \begin{array}{rcl} x_1 + x_3 & = & a \\ 2x_1 + 3x_2 + 8x_3 & = & b \\ 3x_1 - x_2 + x_3 & = & c \end{array} \right\}$. What condition must be given for $a, b, c$ in order that this system of equations be consistent ? Comment on how this coincides with your work in the preceding problem.
*Remark: Lay,* §1.1#25 *is the same sort of problem.*

**Problem 9:** Lay, §1.1#28, question about condition needed on coefficient matrix in order for for the system to be consistent for arbtrary inhomogeneous term.

**Problem 10:** You are given the following row-reduction:

$$\text{rref}\begin{bmatrix} 1 & 2 & 7 & 7 & 3 \\ 2 & 4 & 1 & 0 & 6 \\ 3 & 6 & 3 & 13 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

In view of the given reduction,

**(a.)** provide a consistent system of three equations and three unknowns $x, y, z$ and provide its solution.

**(b.)** provide an inconsistent system of three equations and three unknowns $x, y, z$ and provide its solution.

**(c.)** write down a system of three equations in $x_1, x_2, x_3, x_4$ and provide its solution.

**(d.)** write down a homogeneous system of three equations in $x_1, x_2, x_3, x_4, x_5$ and provide its solution.

**Problem 11:** Lay, §1.1#34, an applied linear system based on temperature distribution.

**Problem 12:** Techniques for solving linear equations also apply to nonlinear equations if they allow a nice substitution. Make a $x_1 = x^2$ and $x_2 = y^2$ substitution to solve:

$$\begin{aligned} x^2 + y^2 &= 4 \\ x^2 - y^2 &= 1 \end{aligned}$$

How many solutions are in the solution set ? Does this make sense graphically ? Make a quick sketch of what is going on here.

**Problem 13:** Solve $\cos\theta - \sin\beta = 1/2$ and $\cos\theta + \sin\beta = 1/3$ given that $-\pi/2 \leq \theta, \beta \leq \pi/2$

**Problem 14:** Interpolation problem:

**(a.)** Find $f(x) = Ax^3 + Bx^2 + Cx + D$ for which $(0, 4), (1, 8), (2, 24), (-1, 0) \in \text{graph}(f)$.

**(b.)** Find the set of all cubic polynomials whose graphs contain $(1, 2)$ and $(-1, 2)$.

**Problem 15:** I suggest using a mixture of row-reduction and back-substitution to solve the following system.

$$
\begin{aligned}
x_1 + x_2 + x_3 &= 6 \\
x_1 + 2x_2 + 3x_3 &= 14 \\
x_1 + 2x_2 + 4x_3 &= 17 \\
2x_3 - 4x_4 - 5x_5 - 6x_6 &= -71 \\
-x_3 + 4x_4 + 6x_5 + 7x_6 &= 85 \\
-x_3 + 4x_4 + 7x_5 + 5x_6 &= 78
\end{aligned}
$$