MATH 221 MISSION 1: LINEAR EQUATIONS AND THE ROW REDUCTION TECHNIQUE (40+5)PTS)

Show your work and box answers. This pdf should be printed single-sided and your answers should be handwritten on the printout with work shown on paper attached after these problem sheets (the work for such problems is to be shown on single-sided paper with no fuzzy edges and is to be included immediate after the problem statement which it supports). However, if the problem says "show work below" then the complete answer is to be given on these problem sheets. Once complete, please staple in upper left corner. Thanks. There are 5pts to earn by completely following the formatting instructions.

Resources This homework is based on Lectures 1 - 4. Also, the following are helpful:

(a.) Chapters 1 & 2 of my lecture notes for Math 221

(b.) Matlab or see http://www.math.odu.edu/ bogacki/cgi-bin/lat.cgi?c=rref
Note: answers and work must be shown in handwritten detailed work unless the problem says "use technology" which is my way of telling you that you can omit routine steps.)

Problem 1: (1pt) Write down the augmented coefficient matrix $[A|b]$ for each system given below:

(a.) $$\left\{ \begin{array}{rcl} x_1 + x_2 + 3x_3 + 4x_4 &=& 12 \\ 2x_2 + 4x_3 - 2x_4 &=& 16 \\ 3x_1 - x_2 + 6x_3 + x_4 &=& 19 \end{array} \right\}$$

(b.) $$\left\{ \begin{array}{rcl} 2x_1 + x_2 + x_3 &=& 0 \\ 3x_1 - x_2 + 3x_3 &=& 0 \\ x_1 + 4x_2 &=& 0 \end{array} \right\}$$

(c.) $$\left\{ \begin{array}{rcl} x_1 - 2x_2 + 3x_3 - 6x_4 &=& 1 \\ 2x_1 - 7x_2 + x_3 + x_4 &=& 2 \end{array} \right\}$$

(d.) $$\left\{ \begin{array}{rcl} 3x_4 + x_6 &=& 1 \\ x_1 - 3x_3 - 5x_5 - 6x_6 &=& 0 \end{array} \right\}$$

Problem 2: (4pt) Use the row-reduction technique to calculate rref$[A|b]$ for each system given in the previous problem and write down the solution set in standard form for each system.

(a.) ______________________________.

(b.) ______________________________.

(c.) ______________________________.

(d.) ______________________________.

Problem 3: (8pt) Calculate rref$(A|b)$ and find the solution set in standard form for $Ax = b$

(a.) If $A = \begin{bmatrix} 0 & 3 & 2 \\ 1 & 1 & 1 \\ 3 & 8 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 19 \\ 0 \\ -38 \end{bmatrix}$ then rref$(A|b) =$ ____________________.

and the solution set is: ____..

(b.) If $A = \begin{bmatrix} 0 & 3 & 2 & 19 \\ 1 & 1 & 1 & 0 \\ 3 & 8 & 0 & -38 \end{bmatrix}$ and $b = 0$ then rref$(A|b) =$ ____________________.

and the solution set is: ____..

(c.) If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 4 & 2 \\ 3 & 3 & 0 \\ 4 & 4 & 0 \end{bmatrix}$ and $b = 0$ then rref$(A|b) =$ ____________________.

and the solution set is: ____..

(d.) If $A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 20 \\ 13 \end{bmatrix}$ then rref$(A|b) =$ ____________________.

and the solution set is: ____..

Problem 4: (1pt) Let $a \in \mathbb{R}$. Find the solution set in standard form for $ax + y = 1$ and $x + y = 2$. Break into cases as needed.

Problem 5: (1pt) Find the solution in standard form. Hint: divide and conquer.

$$\begin{aligned} x_1 + x_2 + x_3 &= 6 \\ x_1 + 2x_2 + 3x_3 &= 14 \\ x_1 + 2x_2 + 4x_3 &= 17 \\ 2x_3 - 4x_4 - 5x_5 - 6x_6 &= -71 \\ -x_3 + 4x_4 + 6x_5 + 7x_6 &= 85 \\ -x_3 + 4x_4 + 7x_5 + 5x_6 &= 78 \end{aligned}$$

Problem 6: (3pt) Consider the following systems:

$$\textbf{(I.)} \quad \left\{ \begin{array}{rcl} x_1 + x_3 & = & 1 \\ 2x_1 + 3x_2 + 8x_3 & = & 1 \\ 3x_1 - x_2 + x_3 & = & -4 \end{array} \right\} \qquad \& \qquad \textbf{(II.)} \quad \left\{ \begin{array}{rcl} x_1 + x_3 & = & 1 \\ 2x_1 + 3x_2 + 8x_3 & = & 5 \\ 3x_1 - x_2 + x_3 & = & 2 \end{array} \right\}$$

These systems share the same matrix of coefficients A whereas the inhomogeneous terms differ. If $[A|b_I]$ is the augmented coefficient matrix for **(I.)** and $[A|b_{II}]$ is the augmented coefficient matrix for **(II.)** then we may solve both systems by row-reducing $[A|b_I|b_{II}]$.

(a.) use technology to calculate rref$[A|b_I|b_{II}]$

(b.) write down the standard solution to system **(I.)**, or state no solution.

(c.) write down the standard solution to system **(II.)**, or state no solution.

Problem 7: (1pt) Let a, b, c be constants. Consider $\left\{ \begin{array}{rcl} x_1 + x_3 & = & a \\ 2x_1 + 3x_2 + 8x_3 & = & b \\ 3x_1 - x_2 + x_3 & = & c \end{array} \right\}$. What condition must be given for a, b, c in order that this system of equations be consistent ? Comment on how this coincides with your work in the preceding problem.

Problem 8: (1pt) Techniques for solving linear equations also apply to nonlinear equations if they allow a nice substitution. Make a $x_1 = x^2$ and $x_2 = y^2$ substitution to solve:

$$\begin{array}{rl} x^2 + y^2 & = 4 \\ x^2 - y^2 & = 1 \end{array}$$

How many solutions are in the solution set ? Does this make sense graphically ? Make a quick sketch of what is going on here.

Problem 9: (1pt) Solve $\cos\theta - \sin\beta = 1/2$ and $\cos\theta + \sin\beta = 1/3$ given that $-\pi/2 \leq \theta, \beta \leq \pi/2$

Problem 10: (4pt) You are given the following row-reduction:

$$\text{rref}\begin{bmatrix} 1 & 2 & 7 & 7 & 3 \\ 2 & 4 & 1 & 0 & 6 \\ 3 & 6 & 3 & 13 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

In view of the given reduction,

(a.) provide a consistent system of three equations and three unknowns x, y, z and provide its solution.

(b.) provide an inconsistent system of three equations and three unknowns x, y, z and provide its solution.

(c.) write down a system of three equations in x_1, x_2, x_3, x_4 and provide its solution.

(d.) write down a homogeneous system of three equations in x_1, x_2, x_3, x_4, x_5 and provide its solution.

Problem 11: (2pt) Interpolation problem: use technology for row-reductions to solve the following:

(a.) Find $f(x) \in P_3(\mathbb{R})$ for which $(1, 13), (2, 62), (3, 183), (-1, -13) \in \text{graph}(f)$.

(b.) Find the set of all cubic polynomials whose graphs contain $(0, -1)$ and $(0, 7)$. Express your answer using span notation.

Problem 12: (1pt) Let a, b be constants. Solve $\left\{ \begin{array}{ll} x & = a - 2y \\ 4y & = b - 3x \end{array} \right\}$. Your answer will involve a and b.

Problem 13: (1pt) Solve $x\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + y\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + z\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$.

Problem 14: (1pt) Solve $x(3, 2, 1, -1) + y(0, 1, -2, 2) + z(3, 3, -1, 1) = 0$.

Problem 15: (2pt) Consider $S = \{(1, 2, 2, 4), (0, 1, 1, 2), (6, 19, 19, 38)\}$

(a.) Is S a LI set ? If not, provide an explicit linear dependence of the vectors in S.

(b.) Use technology to decide if $e_j \in span(S)$ for $j = 1, 2, 3, 4$. Here $e_1 = (1, 0, 0, 0)$ and $e_2 = (0, 1, 0, 0)$ and $e_3 = (0, 0, 1, 0)$ and $e_4 = (0, 0, 0, 1)$.

Problem 16: (2pt) Let $f(x)$ be a real polynomial such that $f'''(x) = 0$ and $(2, 0), (3, 0) \in graph(f)$. Write the set of all such polynomials as a span.

Problem 17: (1pt) Suppose $2\sin x - \cos y + 3\tan z = 3$ and $4\sin x + 2\cos y - 2\tan z = 2$ and $6\sin x - 3\cos y + \tan z = 9$. Given $0 \leq x, y \leq 2\pi$ and $0 \leq z < \pi$, solve this system of nonlinear equations. Feel free to use technology for any row-reductions here.

Problem 18: (1pt) Balance the chemical reaction $C_6H_{12}O_6 \to CO_2 + C_2H_5OH$

Problem 19: (1pt) Let $X_n = (c_n, s_n)$ where c_n is the number of people in the city and s_n is the number of people living in the suburbs at the beginning of year n. Further suppose that $X_{n+1} = AX_n$ where $A = \begin{bmatrix} 0.85 & 0.03 \\ 0.15 & 0.97 \end{bmatrix}$ describes how people move from the city to the suburbs and vice-versa. Given $X_0 = (400, 300)$ use technology to calculate X_1, X_2, X_{10} and X_{100} to one decimal place.

Problem 20: (3pt) Let $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{n\times q}$. Use index notation to carefully prove $(AB)^T = B^T A^T$ (show work below)