Show your work and box answers. This pdf should be printed and your solution should be handwritten on the printout. Once complete, please staple in upper left corner. Thanks.

Suggested Reading You may find the following helpful resources beyond lecture,

(a.) Chapter 2 of my lecture notes for Math 221

(b.) §1.3, 1.4, 1.5, 1.6, 1.7, 2.1 of Lay's *Linear Algebra*

Problem 16: Let $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$. Calculate $AB - BA$. Do A and B commute ?

Problem 17: Let $v = [1, 2, 3, 4] \in \mathbb{R}^{1\times 4}$ and $w = (5, 6, 7, 8) = \begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix} \in \mathbb{R}^{4\times 1}$. Calculate vw and wv.

Problem 18: A matrix is said to be **antisymmetric** if $A^T = -A$. In terms of components, that means $A_{ji} = -A_{ij}$ for all i, j. Consider the 3×3 matrix $A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$. If $\text{Col}_1(A) = \begin{bmatrix} 0 \\ 2 \\ 7 \end{bmatrix}$ and $\text{Row}_3(A) = [7, 3, 0]$ then find A.

Problem 19: Let $N = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. Calculate N^2 and N^3 and show $N^k = 0$ for $k \geq 4$.

Problem 20: Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{bmatrix}$ and $M = \begin{bmatrix} 1 & 0 \\ 3 & -1 \\ 4 & 1 \\ 5 & 2 \end{bmatrix}$. If possible calculate the matrix quantities below. If not, explain why such a calculation does not fall under our definition of matrix addition or multiplication.

(a) $A + M$

(b) A^2

(c) AM

(d) MA

(e) $AA^T + 2MM^T$

(f) $A^TA + 2M^TM$

Problem 21: Let $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ a & b & c & d \end{bmatrix}$ where a, b, c, d are constants. Let e_i denote the i-th standard basis vector for $\mathbb{R}^4$ where $i = 1, 2, 3, 4$. For example, $e_2 = (0, 1, 0, 0) \in \mathbb{R}^{4\times 1} = \mathbb{R}^4$. Also, let $\bar{e}_j$ denote the j-th standard basis element for $\mathbb{R}^3 = \mathbb{R}^{3\times 1}$. For example, $\bar{e}_2 = (0, 1, 0)$. With this notation in mind, calculate:

(a) Ae_2

(b) $\bar{e}_3^T A$

(c) $A[e_1|e_2]$

(d) $\left[\begin{array}{c} \bar{e}_2^T \\ \hline \bar{e}_3^T \end{array}\right] A$

(e) $\bar{e}_3^T A e_2$

Problem 22: Let $X = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$ and $A = \begin{bmatrix} 1 & -2 \\ 3 & 1 \end{bmatrix}$. Solve $AX = I$ where $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the 2×2 identity matrix. Does your solution also solve $XA = I$?

Problem 23: Lay §1.5#6 and §1.5#16 (writing solution sets in vector parametric form)

Problem 24: Let $v_1 = (1, 2, 2)$ and $v_2 = (0, 1, -1)$.

(a.) what condition must be given for a, b, c for $(a, b, c) \in \text{span}(v_1, v_2)$?

(b.) what condition is needed for $\{v_1, v_2, (a, b, c)\}$ to by a linearly independent (LI) set ?

Problem 25: Lay §1.6#5 (solving chemical equation)

Problem 26: Suppose $Av = b$ and $Aw = b$ for a given matrix A and column vector b. If $v \neq w$ then show that the columns of A are linearly dependent.

Problem 27: Let $A = \begin{bmatrix} 1 & 2 & 3 \\ a & b & c \\ 3 & 2 & 1 \end{bmatrix}$ and $R_1 = \begin{bmatrix} a & b & c \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ and $R_2 = \begin{bmatrix} 1 & 2 & 3 \\ a & b & c \\ 2 & 0 & -2 \end{bmatrix}$. Notice both R_1 and R_2 are obtained by performing an elementary row operation on A. Find 3×3 matrices E_1 and E_2 for which $R_1 = E_1 A$ and $R_2 = E_2 A$.

Problem 28: If $A \to R$ under a row-operation then there exists an elementary matrix E for which $R = EA$. Find elementary matrices $E_1, E_2, \dots, E_k$ for which $\text{rref}(A) = E_k E_{k-1} \dots E_2 E_1 A$ for

(a) $k = 3$ will suffice for $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \end{bmatrix}$

(b) $k = 2$ will suffice for $A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 0 & 0 \end{bmatrix}$

Problem 29: We defined the matrix product of $A \in \mathbb{R}^{m \times p}$ with $B \in \mathbb{R}^{p \times n}$ by $(AB)_{ij} = \sum_{k=1}^{p} A_{ik}B_{kj}$ for all $1 \leq i \leq m$ and $1 \leq j \leq n$. In the case $m = n$ we note the matrix $AB \in \mathbb{R}^{n \times n}$. Let us define the **trace** of a square matrix M to be the sum of its diagonals. To be precise, $\text{trace}(M) = \displaystyle\sum_{i=1}^{n} M_{ii}$.

(a) Let I be the $n \times n$ identity matrix. Calculate $\text{trace}(I)$.

(b) Suppose $A \in \mathbb{R}^{n \times p}$ and $B \in \mathbb{R}^{p \times n}$. Show $\text{trace}(AB) = \text{trace}(BA)$.

Problem 30: Let $A = \begin{bmatrix} 1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$. Use Matlab, to calculate $(AA^T)^{100}$ and $(A^TA)^{100}$.

First student who emails me well-documented code to perform these calculations earns 4pts bonus provided I recieve the email at least 3 days before this assignment is due.