Show your work and box answers. This pdf should be printed and your solution should be handwritten on the printout. Once complete, please staple in upper left corner. Thanks.

Suggested Reading You may find the following helpful resources beyond lecture,

(a.) Chapter 2 and §9.1 of my lecture notes for Math 221

(b.) §1.4, 1.5, 1.6, 1.7, 2.1, 2.2, 2.3, 2.4, 2.5 of Lay's *Linear Algebra*

Problem 31: Consider $v = (1, 2, 3, 4)$ and $w = (0, 1, 1, 0)$. Determine if $b_1 = (2, 3, 5, 8) \in \text{span}(v, w)$. Is $b_2 = (1, 0, 0, 0) \in \text{span}(v, w)$? Calculate $\text{rref}[v|w|b_1|b_2]$ and use the CCP (column correspondence property) to answer the questions.

Problem 32: Suppose that $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 1 & 0 & 4 \\ 0 & 1 & -2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Given that $\text{col}_1(A) = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$ and $\text{col}_3(A) = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 3 \end{bmatrix}$ and $\text{col}_4(A) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, use the CCP to find A.

Problem 33: Let $A = \begin{bmatrix} 1 & 9 & 4 & 14 & 8 \\ 2 & 2 & 2 & 6 & 0 \\ 3 & 1 & 0 & 4 & -2 \end{bmatrix}$.

(a.) Calculate $\text{rref}(A)$ and use the CCP to write each non-pivot column of A as a linear combination of the pivot columns

(b.) Find a basis for $\text{Null}(A) = \{x \in \mathbb{R}^5 \mid Ax = 0\}$ and express an arbitary element of the nullspace of A as a linear combination of the basis

Problem 34: Let $A = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 0 & 1 \\ 0 & 3 & 0 \end{bmatrix}$. Calculate A^{-1} and solve $Ax = (a, b, c)$ where a, b, c are constants.

Problem 35: Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$. Calculate A^{-1}.

Problem 36: Let $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. Calculate A^{-1} and solve $Ax = (1, 2, 3, 4, 5)$.

Problem 37: Given that $A^{-1} = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 7 \end{bmatrix}$, calculate $(A^T B)^{-1}$.

Problem 38: Consider a 5×5 matrix A along with vectors $v_1, v_2, v_3, v_4, v_5 \in \mathbb{R}^5$ for which:

$$Av_1 = e_1+e_2, \;\; Av_2 = e_1-e_2, \;\; Av_3 = e_5, \;\; Av_4 = \cos\theta e_3+\sin\theta e_4, , \;\; Av_5 = -\sin\theta e_3+\cos\theta e_4$$

where $e_1 = (1, 0, 0, 0, 0)$ and $e_5 = (0, 0, 0, 0, 1)$ etc. Find the formula for A^{-1} in terms of the given vectors v_1, v_2, v_3, v_4, v_5.

Problem 39: Solve $13x - 2y = a$ and $x - 7y = b$ for arbitrary a, b using matrix techniques. I recommend multiplication by inverse of the coefficient matrix.

Problem 40: Let $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 3 \\ 2 & 0 & k \end{bmatrix}$. For what values of k does A^{-1} exist ?

Problem 41: Lay §2.4#9. Symbolic block matrix algebra problem.

Problem 42: Let $M = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \in \mathbb{R}^{4\times 4}$ where $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \cos\theta & \sin\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cosh\phi & \sinh\phi \\ \sinh\phi & \cosh\phi \end{bmatrix}$. Calculate A^{-1} and B^{-1} via the 2×2 inverse formula then check, via block-multiplication of the partitioned matrix M that $M^{-1} = \begin{bmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{bmatrix}$.

Problem 43: If $v = (a, b, c)$ and $w = (x, y, z)$ are two vectors in $\mathbb{R}^3$ then the **dot-product** of v and w is given by $v \bullet w = v^T w = ax + by + cz$ and the **length** of the vector v is given by $\|v\| = \sqrt{v \bullet v} = \sqrt{a^2 + b^2 + c^2}$. Suppose we are given vectors of length one which are pairwise-perpendicular; that is $u_1 \bullet u_2 = 0$ and $u_1 \bullet u_3 = 0$ and $u_2 \bullet u_3 = 0$. Let $M = [u_1|u_2|u_3]$. Calculate $M^T M$ and simplify in view of the given information. Calculate M^{-1} in terms of u_1, u_2, u_3.

Problem 44: Lay §2.5#3. Solution aided by a given LU-decomposition.

Problem 45: Let $A = \begin{bmatrix} 2 & -4 & 4 & -2 \\ 6 & -9 & 7 & -3 \\ -1 & -4 & 8 & 0 \end{bmatrix}$. Find the LU-decomposition for A. I believe this matrix does not require the introduction of a permutation matrix like I faced in the example solved https://math.stackexchange.com/a/186997/36530. There is a natural algorithm which helps us construct the LU-decomposition from following the forward-pass of the row-reduction on A. (this is Lay §2.5#15 but I don't think the slick method outlined in my linked answer is to be found in Lay)