Show your work and box answers. This pdf should be printed and your solution should be handwritten on the printout. Once complete, please staple in upper left corner. Thanks.

Suggested Reading You may find the following helpful resources beyond lecture,

(a.) Chapter 3 of my lecture notes for Math 221

(b.) Chapter 3 of Lay's *Linear Algebra*

Problem 46: Calculate $\det(A)$ where $A = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 2 & 2 \\ 5 & 3 & 1 \end{bmatrix}$

Problem 47: Calculate $\det(B)$ where $B = \begin{bmatrix} 2 & 2 & 0 & 2 & 2 \\ 0 & 2 & 0 & -1 & 0 \\ 2 & 2 & 0 & 0 & 3 \\ 7 & 7 & 2 & 7 & 7 \\ 5 & 3 & 0 & 0 & 0 \end{bmatrix}$

Problem 48: Let A, B be as given in the previous problems. If $M = \left[\begin{array}{c|c} 2A & 0 \\ \hline 0 & 3B \end{array}\right]$ then calculate $\det(M)$ via application of properties of determinants given in the lecture notes and the results of the previous pair of problems.

Problem 49: For which values of x is the matrix $M = \begin{bmatrix} x & 2 & 2 \\ 1 & 1 & 1 \\ 7 & 5 & 3 \end{bmatrix}$ invertible?

Problem 50: Solve $\alpha x + 3y = 7$ and $5x - \beta y = 6$ by Cramer's rule. Comment on needed conditions on α, β for the solution to exist.

Problem 51: Let A be a matrix which is similar to $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$. In other words, suppose there exists an invertible matrix P for which $B = P^{-1}AP$. Calculate $\det(A)$ and $\text{trace}(A)$.

Problem 52: Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$. Calculate $\det(A)$ using properties of the determinant based on row-reductions. Almost certainly using Laplace's expansion by minors here is a really bad idea.

Problem 53: Find the volume of the paralell piped with edges $(1, 2, 3), (2, 3, 3), (-1, -2, 0)$.

Problem 54: Lay, §3.2#40 (problem to help learn properties of determinant)

Problem 55: Lay, Chapter 3, page 212 #7 on (equation of line)

Problem 56: Lay, §3.3#6 (Cramer's rule)

Problem 57: Is $(a, b, c) \in \text{span}\{(1, 2, 3), (0, 1, 1)\}$? Use determinants and the theory of linear algebra we have discussed to answer this question.

Problem 58: Suppose you have a square matrix A for which the matrix equation $A^T JA = J$ holds for some invertible matrix J. Find the possible values for $det(A)$.

Problem 59: **The cross product:** For all $a, b \in \mathbb{R}^3$ we define

$$T(a,b) = \sum_{j=1}^{3} \big(\det[a|b|e_j]\big) e_j.$$

Show $a \bullet T(a,b) = 0$ and $b \bullet T(a,b) = 0$ and for any $c \in \mathbb{R}^3$ we have $T(a,b) \bullet c = \det[a|b|c]$.

Problem 60: A natural candidate for the cross product in $\mathbb{R}^4$ is given by extending the formula in the previous problem: for all $a, b, c \in \mathbb{R}^4$ we define

$$T(a, b, c) = \sum_{j=1}^{4} \big(\det[a|b|c|e_j]\big) e_j$$

Show: $a \bullet T(a, b, c) = 0$ and $b \bullet T(a, b, c) = 0$ and $c \bullet T(a, b, c) = 0$.
I should mention, the equations above tell us a, b, c are perpendicular to $T(a, b, c)$ and we can prove that implies $\{a, b, c, T(a, b, c)\}$ is linearly independent provided $T(a, b, c) \neq 0$. In other words, if you want a fourth vector which is outside the span of $a, b, c \in \mathbb{R}^4$ then $T(a, b, c)$ is a nice choice. It is the normal to the hypervolume spanned by a, b, c.