

Show your work and box answers. This pdf should be printed and your solution should be handwritten on the printout. Once complete, please staple in upper left corner. Thanks.

**Suggested Reading** You may find the following helpful resources beyond lecture,

(a.) Chapter 7 and §9.2 of my lecture notes for Math 221

(b.) Chapter 6 and §7.1, 7.2 of Lay's *Linear Algebra*

**Problem 91:** Consider the quadratic form given by

$$Q(x, y) = x^2 + 4xy + y^2$$

Find the diagonalized formula for  $Q$  in terms of eigencoordinates  $\bar{x}, \bar{y}$ .

**Problem 92:** Consider the quadratic form given by

$$Q(x, y, z) = 9x^2 + 9y^2 + 29z^2 + 3.5xy - 6.5xz - 6.5yz.$$

Find the diagonalized formula for  $Q$  in terms of eigencoordinates  $\bar{x}, \bar{y}, \bar{z}$ .

**Problem 93:** Consider a subspace  $W$  of  $\mathbb{R}^4$  which contains the vectors  $(1, 1, 2, 3)$  and  $(1, 0, 4, 5)$ . Find a basis for  $W^\perp$ .

**Problem 94:** Find an orthonormal basis for

$$W = \text{span}\{(1, 1, 1, 1), (0, 1, -1, 1), (2, 0, 2, 0)\}$$

by using the Gram-Schmidt algorithm on the given generating vectors. Also, find an orthonormal basis for  $W^\perp$ .

**Problem 95:** Let  $f(x, y, z) = 9x^2 + 14 \sin(xy) + y \sinh y + 29e^{z^2} - 26z(x + y)$ .

- (a.) Calculate partial derivatives  $f_x, f_y, f_z, f_{xx}, f_{xy}, f_{xz}, f_{yy}, f_{yz}, f_{zz}$
- (b.) calculate the multivariate Taylor series based at  $(0, 0, 0)$  up to second order. You should find that  $(0, 0, 0)$  is a critical point hence  $f(x, y, z) = f(0, 0, 0) + Q(z, y, z) + \dots$  where the quadratic form  $Q$  has matrix with entries fixed by the values of the second derivatives of  $f$  at  $(0, 0, 0)$ :

$$[Q] = \begin{bmatrix} f_{xx}(0, 0, 0) & f_{xy}(0, 0, 0) & f_{xz}(0, 0, 0) \\ f_{xy}(0, 0, 0) & f_{yy}(0, 0, 0) & f_{yz}(0, 0, 0) \\ f_{xz}(0, 0, 0) & f_{yz}(0, 0, 0) & f_{zz}(0, 0, 0) \end{bmatrix}$$

- (c.) classify the nature of the critical point  $(0, 0, 0)$  by diagonalizing  $Q$ . Is the function minimized, maximized or is it at a saddle point at the origin ?

*Remark: you stumble across a homeless mathematician with chalk in hand scribbling on the ground, you notice the phrase 78 WAS A GOOD YEAR, strange, what does this mean.*

Feel free to work these on your own paper, but be sure they are labeled as indicated below and put in order neatly. Thanks!

**Problem 96:** Lay §6.2#8, 10

**Problem 97:** Lay §6.2#14, 15

**Problem 98:** Lay §6.3#4

**Problem 99:** Lay §6.3#12, 16

**Problem 100:** Lay §6.4#11, 15 (Gram-Schmidt and QR-factorization)

**Problem 101:** Lay §6.5#4, 8

**Problem 102:** Lay §6.5#11

**Problem 103:** Lay §6.5#15

**Problem 104:** Lay §6.6#1

**Problem 105:** Lay §6.6#7