Show your work and box answers. Once complete, please staple in upper left corner. Thanks.

Suggested Reading You may find the following helpful resources beyond lecture,

(a.) Chapter 1 of my lecture notes for Math 221

(b.) Reduced Row Echelon Web Calculator
(see http://www.math.odu.edu/ bogacki/cgi-bin/lat.cgi?c=rref
please consider using this to check your answers, or better yet, use Matlab, but the answers and work must be shown in handwritten detailed work here. To be clear, you cannot just use Matlab and printout the result. Not yet anyway, all in good time.)

Problem 1: Solve the following system by back-substitution. In particular, solve the last equation and use the solution to simplify the preceding equation, rinse and repeat.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 &= 55 \\ 2x_2 + 3x_3 + 4x_4 + 5x_5 &= 54 \\ 3x_3 + 4x_4 + 5x_5 &= 50 \\ 4x_4 + 5x_5 &= 41 \\ 5x_5 &= 25 \end{aligned}$$

Problem 2: Solve each system and graph each equation. Explain why the system is inconsistent for the inconsistent systems.

(a.) $y = x + 2$ and $y = -x + 3$

(b.) $2x + 3y = 1$ and $4x + 6y = 2$

(c.) $y = x + 1$ and $y = 1 - x$ and $2x - 2y = 4$

Problem 3: Let a, b be constants. Solve $\left\{ \begin{array}{ll} x + 2y & = a \\ 3x + 4y & = b \end{array} \right\}$. Your answer will involve a and b.

Problem 4: Solve the following system of equations via row-reduction of the augmented coefficient matrix.

$$\left\{ \begin{array}{rcl} x + 2y &=& 3z - w \\ 2x - y + z &=& w + 2 \\ y + z + w &=& 6 \\ 3z + x &=& 12 + y - w \end{array} \right\}$$

Problem 5: Write down the augmented coefficient matrix $[A|b]$ for each system given below:

(a.) $$\left\{ \begin{array}{rcl} x_1 + x_2 + 3x_3 + 4x_4 &=& 12 \\ 2x_2 + 4x_3 - 2x_4 &=& 16 \\ 3x_1 - x_2 + 6x_3 + x_4 &=& 19 \end{array} \right\}$$

(b.) $$\left\{ \begin{array}{rcl} 2s + t &=& 1 \\ 3s - t &=& 3 \\ s + 4t &=& 0 \end{array} \right\}$$

$$\textbf{(c.)} \left\{ \begin{array}{rl} 2x_1 + x_2 + x_3 &= 0 \\ 3x_1 - x_2 + 3x_3 &= 0 \\ x_1 + 4x_2 &= 0 \end{array} \right\}$$

$$\textbf{(d.)} \left\{ \begin{array}{rl} x_1 - 2x_2 + 3x_3 - 6x_4 &= 1 \\ 2x_1 - 7x_2 + x_3 + x_4 &= 2 \end{array} \right\}$$

$$\textbf{(e.)} \left\{ \begin{array}{rl} 3x_4 + x_6 - x_7 &= 1 \\ 2x_1 - 2x_2 - 3x_3 - 4x_4 - 5x_5 - 6x_6 - 7x_7 &= 0 \end{array} \right\}$$

Problem 6: Use the row-reduction technique to calculate rref$[A|b]$ for each system given in the previous problem and write down the solution set in standard form for each system. (attach the solutions in order, clearly labeled after this page)

Problem 7: Consider the following systems:

$$\textbf{(I.)} \left\{ \begin{array}{rl} x_1 + x_3 &= 1 \\ 2x_1 + 3x_2 + 8x_3 &= 1 \\ 3x_1 - x_2 + x_3 &= -4 \end{array} \right\}$$

$$\textbf{(II.)} \left\{ \begin{array}{rl} x_1 + x_3 &= 1 \\ 2x_1 + 3x_2 + 8x_3 &= 5 \\ 3x_1 - x_2 + x_3 &= 2 \end{array} \right\}$$

These systems share the same matrix of coefficients A whereas the inhomogeneous terms differ. If $[A|b_I]$ is the augmented coefficient matrix for **(I.)** and $[A|b_{II}]$ is the augmented coefficient matrix for **(II.)** then we may solve both systems by row-reducing $[A|b_I|b_{II}]$.

(a.) write down $[A|b_I|b_{II}]$

(b.) use row-reduction to calculate rref$[A|b_I|b_{II}]$

(c.) write down the standard solution to system **(I.)** and **(II.)**, or state no solution.

Problem 8: Let a, b, c be constants. Consider $\left\{ \begin{array}{rl} x_1 + x_3 &= a \\ 2x_1 + 3x_2 + 8x_3 &= b \\ 3x_1 - x_2 + x_3 &= c \end{array} \right\}$. What condition must be given for a, b, c in order that this system of equations be consistent ? Comment on how this coincides with your work in the preceding problem.

Problem 9: Consider constants a, b, c, d with $a \neq 0$. If $ax_1 + bx_2 = f$ and $cx_1 + dx_2 = g$ is consistent for all f, g then find a necessary condition on a, b, c, d.

Problem 10: You are given the following row-reduction:

$$\text{rref} \begin{bmatrix} 1 & 2 & 7 & 7 & 3 \\ 2 & 4 & 1 & 0 & 6 \\ 3 & 6 & 3 & 13 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

In view of the given reduction,

(a.) provide a consistent system of three equations and three unknowns x, y, z and provide its solution.

(b.) provide an inconsistent system of three equations and three unknowns x, y, z and provide its solution.

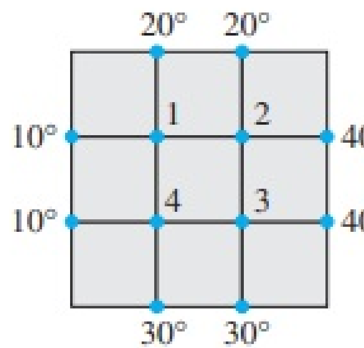
(c.) write down a system of three equations in x_1, x_2, x_3, x_4 and provide its solution.

(d.) write down a homogeneous system of three equations in x_1, x_2, x_3, x_4, x_5 and provide its solution.

Problem 11: Work out the Lay, §1.1#33&34 shown below:

An important concern in the study of heat transfer is to determine the steady-state temperature distribution of a thin plate when the temperature around the boundary is known. Assume the plate shown in the figure represents a cross section of a metal beam, with negligible heat flow in the direction perpendicular to the plate. Let $T_1, \ldots, T_4$ denote the temperatures at the four interior nodes of the mesh in the figure. The temperature at a node is approximately equal to the average of the four nearest nodes—to the left, above, to the right, and below.[3] For instance,

$$T_1 = (10 + 20 + T_2 + T_4)/4, \quad \text{or} \quad 4T_1 - T_2 - T_4 = 30$$



33. Write a system of four equations whose solution gives estimates for the temperatures $T_1, \ldots, T_4$.

34. Solve the system of equations from Exercise 33. [*Hint*: To speed up the calculations, interchange rows 1 and 4 before starting "replace" operations.]

[3] See Frank M. White, *Heat and Mass Transfer* (Reading, MA: Addison-Wesley Publishing, 1991), pp. 145–149.

Problem 12: Techniques for solving linear equations also apply to nonlinear equations if they allow a nice substitution. Make a $x_1 = x^2$ and $x_2 = y^2$ substitution to solve:

$$\begin{aligned} x^2 + y^2 &= 4 \\ x^2 - y^2 &= 1 \end{aligned}$$

How many solutions are in the solution set ? Does this make sense graphically ? Make a quick sketch of what is going on here.

Problem 13: Solve $\cos\theta - \sin\beta = 1/2$ and $\cos\theta + \sin\beta = 1/3$ given that $-\pi/2 \leq \theta, \beta \leq \pi/2$

Problem 14: Interpolation problem:

(a.) Find $f(x) = Ax^3 + Bx^2 + Cx + D$ for which $(0, 4), (1, 8), (2, 24), (-1, 0) \in \text{graph}(f)$.

(b.) Find the set of all cubic polynomials whose graphs contain $(1, 2)$ and $(-1, 2)$.

Problem 15: I suggest using a mixture of row-reduction and back-substitution to solve the following system.

$$\begin{aligned} x_1 + x_2 + x_3 &= 6 \\ x_1 + 2x_2 + 3x_3 &= 14 \\ x_1 + 2x_2 + 4x_3 &= 17 \\ 2x_3 - 4x_4 - 5x_5 - 6x_6 &= -71 \\ -x_3 + 4x_4 + 6x_5 + 7x_6 &= 85 \\ -x_3 + 4x_4 + 7x_5 + 5x_6 &= 78 \end{aligned}$$