Show your work and box answers. Once complete, please staple in upper left corner. Thanks.

Suggested Reading You may find the following helpful resources beyond lecture,

(a.) Chapters 3 and 4 of my lecture notes for Math 221

Problem 31: Consider $v = (1,2,3,4)$ and $w = (0,1,1,0)$. Determine if $b_1 = (2,3,5,8) \in \text{span}(v,w)$. Is $b_2 = (1,0,0,0) \in \text{span}(v,w)$? Calculate $\text{rref}\,[v|w|b_1|b_2]$ and use the CCP (column correspondence property) to answer the questions.

Problem 32: Suppose that $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 1 & 0 & 4 \\ 0 & 1 & -2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Given that $\text{col}_1(A) = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 3 \end{bmatrix}$ and $\text{col}_3(A) = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 3 \end{bmatrix}$ and $\text{col}_4(A) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, use the CCP to find A.

Problem 33: Let $A = \begin{bmatrix} 1 & 9 & 4 & 14 & 8 \\ 2 & 2 & 2 & 6 & 0 \\ 3 & 1 & 0 & 4 & -2 \end{bmatrix}$.

(a.) Calculate $\text{rref}(A)$ and use the CCP to write each non-pivot column of A as a linear combination of the pivot columns

(b.) Find a basis for $\text{Null}(A) = \{x \in \mathbb{R}^5 \mid Ax = 0\}$ and express an arbitary element of the nullspace of A as a linear combination of the basis

Problem 34: Let $A = \begin{bmatrix} 1 & 0 & 2 \\ -2 & 0 & 1 \\ 0 & 3 & 0 \end{bmatrix}$. Calculate A^{-1} and solve $Ax = (a,b,c)$ where a,b,c are constants.

Problem 35: Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$. Calculate A^{-1}.

Problem 36: Let $A = \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. Calculate A^{-1} and solve $Ax = (1,2,3,4,5)$.

Problem 37: Given that $A^{-1} = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} -1 & 1 \\ 1 & 7 \end{bmatrix}$, calculate $(A^TB)^{-1}$.

Problem 38: Consider a 5×5 matrix A along with vectors $v_1, v_2, v_3, v_4, v_5 \in \mathbb{R}^5$ for which:

$$Av_1 = e_1+e_2, \;\; Av_2 = e_1-e_2, \;\; Av_3 = e_5, \;\; Av_4 = \cos\theta e_3+\sin\theta e_4,, \;\; Av_5 = -\sin\theta e_3+\cos\theta e_4$$

where $e_1 = (1,0,0,0,0)$ and $e_5 = (0,0,0,0,1)$ etc. Find the formula for A^{-1} in terms of the given vectors v_1, v_2, v_3, v_4, v_5.

Problem 39: Solve $13x-2y = a$ and $x-7y = b$ for arbitrary a, b using matrix techniques. I recommend multiplication by inverse of the coefficient matrix.

Problem 40: Let $A = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 3 \\ 2 & 0 & k \end{bmatrix}$. For what values of k does A^{-1} exist ?

Problem 41: Consider the block matrix equation $\begin{bmatrix} I & 0 & 0 \\ X & I & 0 \\ Y & 0 & I \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \\ 0 & B_{32} \end{bmatrix}$. Solve for X, Y and B_{22} given that A_{11}^{-1} exists.

Problem 42: Let $M = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} \in \mathbb{R}^{4\times 4}$ where $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cosh\phi & \sinh\phi \\ \sinh\phi & \cosh\phi \end{bmatrix}$. Calculate A^{-1} and B^{-1} via the 2×2 inverse formula then check, via block-multiplication of the partitioned matrix M that $M^{-1} = \begin{bmatrix} A^{-1} & 0 \\ 0 & B^{-1} \end{bmatrix}$.

Problem 43: If $v = (a, b, c)$ and $w = (x, y, z)$ are two vectors in $\mathbb{R}^3$ then the **dot-product** of v and w is given by $v \bullet w = v^T w = ax + by + cz$ and the **length** of the vector v is given by $\|v\| = \sqrt{v \bullet v} = \sqrt{a^2 + b^2 + c^2}$. Suppose we are given vectors of length one which are pairwise-perpendicular; that is $u_1 \bullet u_2 = 0$ and $u_1 \bullet u_3 = 0$ and $u_2 \bullet u_3 = 0$. Let $M = [u_1|u_2|u_3]$. Calculate $M^T M$ and simplify in view of the given information. Calculate M^{-1} in terms of u_1, u_2, u_3.

Problem 44: Notice $A = \begin{bmatrix} 2 & -1 & 2 \\ -6 & 0 & -2 \\ 8 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 4 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \\ 0 & -3 & 4 \\ 0 & 0 & 1 \end{bmatrix}$ is an LU-decomposition. Solve $Ax = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$ using the LU-decomposition.

Problem 45: Let $A = \begin{bmatrix} 2 & -4 & 4 & -2 \\ 6 & -9 & 7 & -3 \\ -1 & -4 & 8 & 0 \end{bmatrix}$. Find the LU-decomposition for A. I believe this matrix does not require the introduction of a permutation matrix like I faced in the example solved https://math.stackexchange.com/a/186997/36530. There is a natural algorithm which helps us construct the LU-decomposition from following the forward-pass of the row-reduction on A.