Show your work and box answers. Once complete, please staple in upper left corner. Thanks.

Suggested Reading You may find the following helpful resources beyond lecture,

(a.) Chapter 9 and 10 of my lecture notes for Math 221

Problem 91: Consider the quadratic form given by

$$Q(x, y) = x^2 + 4xy + y^2$$

Find the diagonalized formula for Q in terms of eigencoordinates $\bar{x}, \bar{y}$.

Problem 92: Consider the quadratic form given by

$$Q(x, y, z) = 9x^2 + 9y^2 + 29z^2 + 3.5xy - 6.5xz - 6.5yz.$$

Find the diagonalized formula for Q in terms of eigencoordinates $\bar{x}, \bar{y}, \bar{z}$.

Problem 93: Consider a subspace W of $\mathbb{R}^4$ which contains the vectors $(1, 1, 2, 3)$ and $(1, 0, 4, 5)$. Find a basis for $W^\perp$.

Problem 94: Find an orthonormal basis for

$$W = \text{span}\{(1, 1, 1, 1), (0, 1, -1, 1), (2, 0, 2, 0)\}$$

by using the Gram-Schmidt algorithm on the given generating vectors. Also, find an orthonormal basis for $W^\perp$.

Problem 95: Let $f(x, y, z) = 9x^2 + 14\sin(xy) + 9y\sinh y + 29e^{z^2} - 26z(x + y)$.

(a.) Calculate partial derivatives $f_x, f_y, f_z, f_{xx}, f_{xy}, f_{xz}, f_{yy}, f_{yz}, f_{zz}$

(b.) calculate the multivariate Taylor series based at $(0, 0, 0)$ up to second order. You should find that $(0, 0, 0)$ is a critical point hence $f(x, y, z) = f(0, 0, 0) + Q(z, y, z) + \cdots$ where the quadratic form Q has matrix with entries fixed by the values of the second derivatives of f at $(0, 0, 0)$:

$$[Q] = \begin{bmatrix} f_{xx}(0,0,0) & f_{xy}(0,0,0) & f_{xz}(0,0,0) \\ f_{xy}(0,0,0) & f_{yy}(0,0,0) & f_{yz}(0,0,0) \\ f_{xz}(0,0,0) & f_{yz}(0,0,0) & f_{zz}(0,0,0) \end{bmatrix}$$

(c.) classify the nature of the critical point $(0, 0, 0)$ by diagonalizing Q. Is the function minimized, maximized or is it at a saddle point at the origin ?

Problem 96: Show the given vectors u_i are orthogonal and express x as a linear combination of the u_i.

(a.) $u_1 = (3, 1)$ and $u_2 = (-2, 6)$ with $x = (-6, 3)$.

(b.) $u_1 = (3, -3, 0)$ and $u_2 = (2, 2, -1)$ and $u_3 = (1, 1, 4)$ with $x = (5, -3, 1)$.

Problem 97: **(a.)** Let $y = (2, 6)$ and $u = (7, 1)$. Write y as a sum of a vector in $span\{u\}$ and a vector $\perp$ to u.

(b.) Let $y = (3, 1)$ and $u = (8, 6)$. Compute the distance from y to the line which goes through u and the origin.

Problem 98: Let $u_1 = (3, -1, 2)$ and $u_2 = (1, -1, -2)$ and suppose $y = (-1, 2, 6)$. Verify $\{u_1, u_2\}$ is an orthogonal set and then find the orthogonal projection of y onto $W = span\{u_1, u_2\}$.

Problem 99: Let $y = (3, -1, 1, 13)$ and $v_1 = (1, -2, -1, 2)$ and $v_2 = (-4, 1, 0, 3)$.

(a.) Find the closest point to y in $W = span\{v_1, v_2\}$

(b.) Find the distance from y to $W = span\{v_1, v_2\}$.

Problem 100: Let $A = \begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}$. Find an orthogonal basis for the column space of A and find the QR-decompositon for A.

Problem 101: Let $A = \begin{bmatrix} 1 & 3 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$. Find the least squares approximate solution of $Ax = b$. Also, calculate the error in the least squares solution.

Problem 102: Let $A = \begin{bmatrix} 4 & 0 & 1 \\ 1 & -5 & 1 \\ 6 & 1 & 0 \\ 1 & -1 & -5 \end{bmatrix}$ and $b = \begin{bmatrix} 9 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

(a.) find $Proj_{Col(A)}(b)$

(b.) find least squares solution of $Ax = b$

Problem 103: Observe $A = \begin{bmatrix} 2 & 3 \\ 2 & 4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 \\ 2/3 & 2/3 \\ 1/3 & -2/3 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 0 & 1 \end{bmatrix}$ is a QR-decomposition of A. Let $b = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$. Use the given QR-decomposition to calculate the least squares approximation to $Ax = b$.

Problem 104: Find β_0, β_1 making $y = \beta_0 + \beta_1 x$ the least squares line for the data points $(0, 1), (1, 1), (2, 2), (3, 2)$.

Problem 105: Suppose the data $(1, 1.8), (2, 2.7), (3, 3.4), (4, 3.8), (5, 3.9)$ is to fit the model $y = \beta_1 x + \beta_2 x^2$.

(a.) substitute the data into the model and obtain a system of linear equations in β_1, β_2

(b.) Find the least squares solution to the system and write the model which corresponds to this solution.