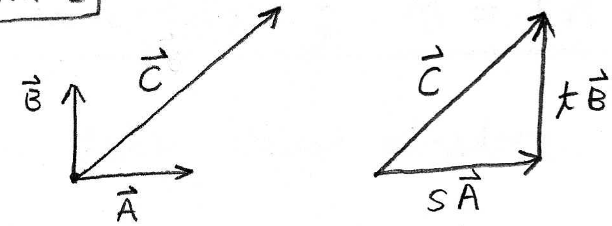
HELP FOR PROBLEMS 1 & 2 OF MATH 231 HOMEWORK PROJECT 1

PROBLEM 1



$s = ?$

$t = ?$

Can use projections to find formulas for s & t.

PROBLEM 2 Projections not helpful since $\vec{A}$ and $\vec{B}$ are not "completely independent", I mean part of $\vec{A}$ can point in the $\vec{B}$-direction. The clean (non geometric) sol$^{\underline{n}}$ is best done with a little basic linear algebra. I'll supply those details here. Ultimately our goal is to find explicit formulas for s and t in terms of c_1, c_2, a, b, c, d where

$$\vec{c} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \qquad \vec{a} = \begin{bmatrix} a \\ b \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} c \\ d \end{bmatrix}.$$

We want s, t such that

$$\vec{C} = s\vec{A} + t\vec{B}$$

we were given that $\vec{A} \not\parallel \vec{B}$. This means there does not exist a k such that $\vec{A} = k\vec{B}$.

$\boxed{\text{Lemma I. } \vec{A} \not\parallel \vec{B} \Rightarrow \vec{A} \text{ and } \vec{B} \text{ are linearly independent}}$

Proof: "linear independence" means $c_1\vec{A} + c_2\vec{B} = 0 \Rightarrow c_1 = c_2 = 0$.
Suppose $\vec{A} \not\parallel \vec{B}$ and $\vec{A}$ and $\vec{B}$ are not linearly independent then $c_1\vec{A} + c_2\vec{B} = 0$ and $c_1 \neq 0$ or $c_2 \neq 0$. Suppose $c_1 \neq 0$ then $\vec{A} + \frac{c_2}{c_1}\vec{B} = 0$ and $\vec{A} = \frac{-c_2}{c_1}\vec{B}$ so there exists $k = -c_2/c_1$ but this is a contradiction.
Hence, by proof by contradiction the Lemma holds true,

Lemma II: If $\vec{A}, \vec{B}$ are linearly independent then the matrix M made by concatenating the vectors is invertible; $M = [\vec{A} | \vec{B}]$ has $\det(M) \neq 0$.

Proof: take linear algebra.

The inverse matrix for $M = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ is given by the formula $\boxed{M^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}}$

The inverse matrix M^{-1} has $M^{-1}M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = $ "1".

- Lets return to the initial problem and see what linear algebra does for us,

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = s\begin{bmatrix} a \\ b \end{bmatrix} + t\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} as + ct \\ bs + dt \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}\begin{bmatrix} s \\ t \end{bmatrix}$$

$$\vec{C} = M\begin{bmatrix} s \\ t \end{bmatrix} \Longrightarrow M^{-1}\vec{C} = M^{-1}M\begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} s \\ t \end{bmatrix}$$

$$\frac{1}{ad-bc}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} s \\ t \end{bmatrix}$$

Multiply the matrix and column vector and that gives us the values of s and t we were looking for. You are free to start with

$s = \dfrac{c_1 d - c_2 c}{ad - bc}$	$t = \dfrac{-c_1 b + c_2 a}{ad - bc}$

plug them into $s\vec{A} + t\vec{B}$ and show you get $\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$.