REVIEW FOR TEST 3 OF CALCULUS III:

The first and best line of defense is to complete and understand the homework and lecture examples. There are two parts to Test 3:

- The **in-class portion of the test will be worth 50%**. Some formulas will be provided, this is a corrected version of the table on page 382, it is taken from a recent version of Taylor's Calculus text, this table will be given for the in-class portion of the test.

Formulas for Grad, Div, Curl, and the Laplacian

	Cartesian (x, y, z) $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are unit vectors in the directions of increasing x, y, and z. F_x, F_y, and F_z are the scalar components of $\mathbf{F}(x, y, z)$ in these directions.	Cylindrical (r, θ, z) $\mathbf{u}_r$, $\mathbf{u}_\theta$, and $\mathbf{k}$ are unit vectors in the directions of increasing r, θ, and z. F_r, F_θ, and F_z are the scalar components of $\mathbf{F}(r, \theta, z)$ in these directions.	Spherical (ρ, ϕ, θ) $\mathbf{u}_\rho$, $\mathbf{u}_\phi$, and $\mathbf{u}_\theta$ are unit vectors in the directions of increasing ρ, ϕ, and θ. F_ρ, F_ϕ, and F_θ are the scalar components of $\mathbf{F}(\rho, \phi, \theta)$ in these directions.
Gradient	$\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial r}\mathbf{u}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\mathbf{u}_\theta + \frac{\partial f}{\partial z}\mathbf{k}$	$\nabla f = \frac{\partial f}{\partial \rho}\mathbf{u}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\mathbf{u}_\phi + \frac{1}{\rho \sin\phi}\frac{\partial f}{\partial \theta}\mathbf{u}_\theta$
Divergence	$\nabla \cdot \mathbf{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$	$\nabla \cdot \mathbf{F} = \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{1}{r}\frac{\partial F_\theta}{\partial \theta} + \frac{\partial F_z}{\partial z}$	$\nabla \cdot \mathbf{F} = \frac{1}{\rho^2}\frac{\partial}{\partial \rho}(\rho^2 F_\rho) + \frac{1}{\rho \sin\phi}\frac{\partial}{\partial \phi}(F_\phi \sin\phi) + \frac{1}{\rho \sin\phi}\frac{\partial F_\theta}{\partial \theta}$
Curl	$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$	$\nabla \times \mathbf{F} = \begin{vmatrix} \frac{1}{r}\mathbf{u}_r & \mathbf{u}_\theta & \frac{1}{r}\mathbf{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ F_r & F_\theta & F_z \end{vmatrix}$	$\nabla \times \mathbf{F} = \begin{vmatrix} \frac{\mathbf{u}_\rho}{\rho^2 \sin\phi} & \frac{\mathbf{u}_\phi}{\rho \sin\phi} & \frac{\mathbf{u}_\theta}{\rho} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial \theta} \\ F_\rho & \rho F_\phi & \rho \sin\phi\, F_\theta \end{vmatrix}$
Laplacian	$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$	$\nabla^2 f = \frac{1}{\rho^2}\frac{\partial}{\partial \rho}\left(\rho^2\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2 \sin\phi}\frac{\partial}{\partial \phi}\left(\sin\phi\frac{\partial f}{\partial \phi}\right) + \frac{1}{\rho^2 \sin^2\phi}\frac{\partial^2 f}{\partial \theta^2}$

- The **take-home portion will also be worth 50%, with a 5pt bonus**. You will get the take-home in class Tuesday. It will cover Green's, Stoke's and the Divergence Theorems, the in-class test will not cover that material directly.

- There is an optional review session in Religion 108 from 5pm-6pm on December 1. I would appreciate it if you would send me an email if you plan to attend, it would be even better if you would let me know what topic you need most help.

Geometry:

- know your curved coordinate systems. It is entirely likely that a question is asked which will require some use of spherical or cylindrical coordinates. Also, the discussion of parametric curves and surfaces in earlier works were to prepare you for this time. In fact, the material on the first test is almost all fair game, we may need to find the equation of a plane as a piece of a bigger problem.

Derivatives:

- know how to calculate ∇f in Cartesians, polars, cylindrical or spherical coordinates.

- know how to calculate $\nabla \times \vec{F}$ in Cartesians, polars, cylindrical or spherical coordinates.

- know how to calculate $\nabla \cdot \vec{F}$ in Cartesians, polars, cylindrical or spherical coordinates.

- know what can and cannot be done with ∇. In other words, understand what the input and outputs are for the curl, divergence and gradient. This is exhaustively studied in the homework problem 17.5#12.

- know what the definition of a conservative vector field. Be able to find a potential function given a conservative vector field. (as in 17.5#13)

- be awake enough to tackle a problem like 17.5#30 or something like it.

- be able to use the cylindrical or spherical coordinate formulas for grad, curl, div or the Laplacian ($\nabla^2 f$). The formulas will be given in the math-type notation.
  (Problem 24 of Homework Project 3 is an example of this sort of thinking)

Line Integrals:

- know how to calculate $\int_C \vec{F} \cdot d\vec{r}$ for a curve in two or three dimensions. Notice this means you need to be well-versed in how to parametrize a curve. Most importantly, be able to parametrize graphs ($y = f(x)$), circles, and line segments. *(I will not put a three dimensional hyperbola or ellipsoid on the test unless I am to provide the parametrization explicitly.)*

- know how the line-integral $\int_C \vec{F} \cdot d\vec{r}$ relates to the integrals $\int_C f\, ds$ and $\int_C P\, dx$, or $\int_C Q\, dy$, or $\int_C R\, dz$. *(see Homework Quiz 13 Solutions)*

- If $\vec{F}$ is a force field then how do you calculate the work done by the force on some particle as it travels the curve C? *(see E141-E142, and E149)*

- If $\vec{F} = \nabla f$ and the curve C goes from the point P to the point Q then how do we calculate $\int_C \vec{F} \cdot \vec{r}$? If I ask you to calculate a line integral it would be wise to check and see if the given vector field is conservative. Especially if the curve is one for which it is hard to find parametric equations. *(see Homework Quiz 14 Solutions, E148)*

Surface Integrals:

- know how to calculate the surface $\int_S \vec{F} \cdot d\vec{S}$ for reasonable surfaces and vector fields. What is reasonable? Problems similar to those covered in lecture or homework of course. For example, if the surface is a graph $z = f(x, y)$, part of a sphere, part of a cylinder, a cube, or perhaps part of a plane. The first thing we must do to calculate a surface integral is to describe the surface parametrically. After that it's just integration. Beware that you may need to use integration techniques we learned in the last part of the course. For example, it is convenient to change to polar coordinates in places. *(see E159, E160, E161, E162, E163 and Homework Quiz 16 Solutions)*

- know how to calculate the scalar surface integral $\int_S f\, dS$. Perhaps I will ask you to calculate the surface area of a sphere, that one wasn't too hard. *(see 401-406 in my notes, homework problems 17.6#37 and 41, and some of Homework Quiz 16)*

Theorems of Vector Calculus:

- know the Fundamental Theorem of Vector Calculus; $\int_C \nabla f \cdot d\vec{r} = f(Q) - f(P)$
  (I assumed the curve began at the point P and ended at the point Q)

- I will test Green's, Stoke's and the Divergence Theorem on the take-home portion of the test. The take-home part will not be super difficult, they will be of similar difficulty to the homework problems. You will have approximately 50.5 hours to complete the take-home. (I need them before I leave school Thursday at 5pm). You may discuss your general idea with your classmates however you should not copy calculations line by line. I do expect that there will be a question on the final based on one of the take-home problems so it would be both unethical and short-sighted to simply copy someone else's work. I trust that you will honor God and your peers in this matter.

Summary of In-class test format:

My goal is to encourage you to nail down the fundamentals of vector calculus. I plan to test the more conceptual/creative side on the take-home test. The line and surface integral calculations do come up in other courses, especially the line integral (in complex variables).

- **Problem 1:** (10pts) Calculate a curl and a divergence, perhaps some combination of both. *[about 15 minutes]*

- **Problem 2:** (15pts) Calculate a line integral. (you will have to find the parametric equations needed) *[about 15 minutes]*

- **Problem 3:** (15pts) Calculate a surface integral. (you will need to find the needed parametric equations for the surface)*[about 30 minutes]*

- **Problem 4:** (10pts) Find the work done by conservative force type-problem
  (if you calculate the line integral here you are not paying attention or just showing off, this problem is meant to be a gift, it shouldn't take more than [5 minutes]).