REVIEW FOR TEST 2 OF CALCULUS III:

The first and best line of defense is to complete and understand the homework and lecture examples. Past that my old test might help you get some idea of how my tests typically look (although our course differs significantly in content, we do ~~not~~ cover Lagrange multipliers or extreme values for closed regions, just local extrema). Most of the test will be like problems you've done before, they may not be the same format but they should require the same skill set. The page numbers on this review refer to my course notes. *(if the chapter numbers are not from 15 or 16 then those problems are from the other version of Stewart, in that version chapter 12 is our chapter 16 essentially. I have solutions to problems from Stewart's Calculus and Concepts posted so I leave those numbers in so you can look at those if you choose. I have placed "hwk soln CC" in front of such numbers to remind you I am referring to my posted solutions of Calculus and Concepts problems.)*

Remark: Your assigned and collected homework plus lecture examples are the most important examples. I mention other solutions for breadth, you should be well-equipped for the test on the basis of the assigned and lectured material alone.

Directional Derivatives: (314-316, look at corrected 314 please)

- know how to calculate ∇f, for a function of two or three Cartesian variables.
- know the definition and interpretation for $D_{\hat{u}}f(p)$ [like Problem 12 on Hwk Proj II].
- what do we demand about $\hat{u}$? (see 15.6 #15 for why this detail matters)
- can you state the formula for $D_{\hat{u}}f(p)$ in terms of ∇f ?
- be aware that you can take directional derivatives for $f(x, y)$ or $f(x, y, z)$ etc…

Tangent planes to graphs and linearizations: (306 and 311)

- how do you find the linearization of f at a point p ? (like E71 on 311, or 15.4#11)
- tangent plane to $z = f(x, y)$ at $(a, b, f(a, b))$ is the graph of the linearization. (15.4#1)
- can you state the formula for the linearization in terms of ∇f ?
- when is a function of two variables differentiable at a point ? (15.4#11)
- what is a critical point for a function of several variables ? (see 15.7#11,13)

- I will give you the min/max theorem (320) if needed. You will still need to know what a critical point is and of course how to calculate f_{xx}, f_{yy}, f_{xy}. (E81 or E82 on 321-322)

- Homework 5 has good problems, however you can ignore section 15.4#39.

- ~~we skipped pages 323-329 which is sad, but true. (skipped 15.7#35)~~ not true. We covered the Extreme Value Th$^{\underline{m}}$ §15.7 #35 is an example which I have a sol$^{\underline{n}}$ posted for in homework 6 sol$^{\underline{n}}$ from Fall 2008

Surfaces: (see homework project II solution for much discussion)

We have yet to cover parametric surfaces

- ~~be able to set-up parametrizations like those in the Homework Project II. In particular, be able to use the cross-section method to pick apart surfaces. This should help you with domains for the parametrizations. The bottom line is that we need some sketch to guide or thinking. It would be good if you thought about how to take apart a 3d-sketch and find useful data by specializing to coordinate planes etc… I would not ask you to draw pretty pictures but I might ask a question that demands some graphical wisdom.~~

- ~~if the homework project solution is too verbose, maybe start with 17.6#19,23 which are solved in my Homework 6 page 1 solution.~~

- know how to use spherical and cylindrical coordinates to your advantage.

- understand what is needed to define a level surface.

- what is the difference between a graph and a level surface?

- ~~what is the difference between a graph and a parametrized surface?~~

- is it possible to describe a surface in more than one way?

- be able to find the tangent plane in each of the ~~three~~ two formalisms. (see 318-319 and ~~Hwk Project II solution, beware some of my solutions are needlessly lengthy)~~

- what is the difference between $\theta = \pi/4$ and $\phi = \pi/4$ geometrically ?

- Homework 5 has good problems, can ignore 15.4#39.

Cartesian Multivariate Integration: [this covers most of pages 330-359].

- Cartesian Topics, sections 16.1, 16.2, 16.3, 16.6 [330-342 my notes]

- double integrals over rectangles (E87-E88) [hwk soln CC 12.2#6,10,12,14,16,23,31]

- triple integrals over boxes (E89-E90) [hwk soln CC 12.7#2 for example]

- double integrals over general regions (E93-E99) [hwk soln CC 12.3#3,10,12,15,19,33,40,43]

- triple integrals over general volumes (E100-E103) [hwk soln CC 12.7#5,6,8,10,14,19]

- be able to calculate bounded volumes. This was done by two methods. $V = \int\int_R f dA$ and $V = \int\int\int_B dV$. How would B and R be related if both represent the same volume V ?

- be able to calculate the area of a region in the xy-plane.

- be able to show that if a region is bounded by $f(x) \geq y \geq g(x)$, $x = a$ and $x = b$ where $a < b$ then the double integral of dA over that region will reproduce the old formula we used back in the earlier part of the calculus sequence. Namely, that $A = \int_a^b (f(x) - g(x))dx$. *(in my course I would have forced you to derive this from an appropriate picture in calculus I or II, but some other instructors might just have given it as a formula to "plug and chug")*

- if another application besides volume/area appears on the test I will give you the formula so you just have to integrate. (like the formulas in E119 or E120 on 355-356)

- what is the difference between $\int\int_R f(x, y)dA$ and $\int_0^1 \int_x^{x^2} f(x, y)dydx$? If these are equal then what is R? Graph R.

- what is the difference between $\int\int\int_B f(x, y, z)dV$ and $\int_0^1 \int_0^1 \int_0^y f(x, y, z)dzdydx$? If these are equal what is B? Sketch B.

- be able to switch bounds when helpful (like in E99).

- homework 7 is a good source of representative test questions.

<u>Coordinate change and integration, [343-359 my notes]</u>

Notice I covered these in an order different then Stewart. I cover 16.9 first so that we can derive the basic formulas we use in 16.4 (double integrals in polars), 16.7(triple integrals in cylindricals) and 16.8(triple integrals in sphericals)

- know the coordinate change theorem and definition of Jacobian on page 344.

- know the definition of Jacobian on page 347 and the theorem on page 349.

- homework 8 has nice problems, however 16.9#7 is not on the test.

- know that $dA = r dr d\theta$ for polars, $dV = r dr d\theta dz$ for cylindricals and $dV = \rho^2 \sin\phi d\rho d\theta d\phi$ for sphericals. Generally, the Jacobian gives the factor that modifies the Cartesian $dA = dxdy$ or $dV = dxdydz$ with these in mind we can construct the theorems, really its just another way of saying the coordinate change theorem. You must change the measure (dA or dV) and substitute everywhere the new variables for the old. Also we must change the bounds, this is just like u-substitution. The wrinkle that might get lost in this way of thinking is that we must order the differentials in the way that makes sense. We must follow the common sense of iterated integrals, complicated bounds on the inside, numeric bounds on the outside. The Theorem on page 349 should be understood in that context.

- I may ask you to work through E109 or E110. Also [hwk soln CC 12.9# 1,4,6.]

- you should be able to convert a region(or volume) from its Cartesian description into its polar coordinate description (or cylindrical or spherical for volumes). This is essential if you are to successfully change integrals from Cartesians to other coordinates.

- double integrals in polar coordinates (E114-116) [hwk from 12.4 hwk soln CC page H78-H80] but start with homework 9 if you don't already understand those.

- triple integrals in cylindrical coordinates (E117) [hwk soln CC 12.8#9,32] and hwk 10

- triple integrals in spherical coordinates (E118-E120)[hwk soln CC 12.8#18], hwk 11.

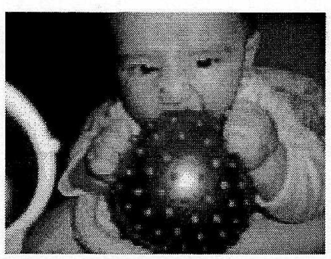
- given an integral in Cartesian coordinates be able to change coordinates wisely. That is have an idea about which coordinate system makes $x^2 + y^2 + z^2$ into something simple, or what about $x^2 + y^2$. [Answer: sphericals and cylindricals or polars respectively].

- we [redacted] covered E108 of 346-347 so that's [redacted] reasonable for the test. (probably not the whole problem but ideas like transforming the region are worthwhile.)

- I expect you to be able to show your work on integrations. I do expect you can do u-substitutions and integrals of sine and cosine squared etc... the integration shouldn't be harder than those that appeared in the required homework.

Main differences:

- no parametric surfaces
- your test may have
  - Extreme Value Th^{m} Problem
  - Lagrange Multiplier Problem (I would remind you the steps, basically walk you through it)



Practice Problems for Test 2

① For understanding coordinates better,

a.) describe the plane $z = 1$ in spherical coordinates.

b.) describe the plane $z = 1$ in cylindrical coordinates.

c.) describe the solid E bounded by

$1 \leq z \leq 2, \quad x^2 + y^2 \leq 9$

in cylindrical coordinates.

d.) desribe E in (c.) in spherical coordinates.

e.) Let $R = \{(x,y) \mid 1 \leq x \leq 2 \text{ and } x \leq y \leq \sqrt{3}x\}$

describe R in polar coordinates.

f.) graph $r = 3\cos(\theta)$ for $0 \leq \theta \leq 2\pi$.

g.) desribe the solid bounded by $z = 3\sqrt{x^2+y^2}$, $x^2+y^2+z^2 = 1$ in spherical coordinates.

② Lagrange Multipliers:

a.) try out $\boxed{E\,85}$ on pg. 327

b.) try out $\boxed{E\,86}$ on pg. 328

I would say "use method of Lagrange Multipliers, recall the central condition for extrema was $\nabla f = \lambda \nabla g$ where $g = 0$ is the constraint and f is the function for which extrema are sought"

③ Find Extreme values for $f(x,y) = x^2\sqrt{x^2+y^2}$ on the the closed disk $D = \{(x,y) \mid x^2+y^2 \leq 1\}$

Notice $\partial D = \{(x,y) \mid x^2+y^2 = 1\} = \{(\cos\theta, \sin\theta) \mid 0 \leq \theta \leq 2\pi\}$

(I like this example because it is not as tedius as the square one we did in lecture.)