Math 231 Mission 1

Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. Your solution is not to be written on the problem sheet. Your solution should stand alone and include a statements which frame the problem. It should not be necessary to read the problem sheet to make sense of your solution. The solution should stand on its own. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks!

Problem 1 Your Printed Name below indicates you have:

- (a.) I have read $\S 1.1 1.4$ of Cook: ______
- (b.) I have attempted 10 problems either from Stewart(see below) or the end of chapter problems in my notes:________.

The following homeworks from Stewart's text are mostly good rudimentary skill problems. These not collected, but, I do recommend you attempt them. I think these¹ problems can be completed in about 2 minutes per problem with the exception of the * problems. As a general rule, I will completely solve the textbook problems in lecture when we have time and you ask. But, you'll get much more out of my solution if you have already attempted it on your own...

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§ 12.1 #'s 15, 17, 23, 25, 29, 37
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§ 12.2 #'s 11, 19, 25, 27, 29, 33, 43, 46*, 47

§ 12.3 #'s 3, 7, 9, 11, 15, 21, 25, 29*, 39, 43, 45, 55

§ 12.4 #'s 1, 3, 7, 13, 15, 20, 27, 31, 33, 39, 43, 48

§ 12.5 #'s 3, 5, 7, 9, 11, 17, 23, 25, 29, 33, 35, 45, 51, 55, 64, 71, 82*

Note: You can be sloppy in the solution of the textbook problems above. That is fine (I am not collecting your textbook problem solutions), but, please present work carefully in the problems which follow. Note also, you do **not** need a copy of Stewart, my notes are self-contained and you can just use the end of chapter problems for additional practice. In addition, there are many old tests and quizzes etc. on the course website.

Problem 2 Find the distance from P = (1, 2, 3) to:

- (a) the point Q = (0, -1, 7)
- (b) the x-axis
- (c) the line through (1, 10, 3) and (2, 2, 2)

¹that is not universal across the later Missions. Don't spend too much time on these, the majority of your time ought to be allocated to the later problems

Problem 3 Let $\vec{A} = \langle 1, 1, 4 \rangle$ and $\vec{B} = \langle 0, 3, 4 \rangle$. Calculate the following:

- (a) A and B
- (b) \widehat{A} and \widehat{B}
- (c) $\vec{A} \cdot \vec{B}$
- (d) $\vec{A} \times \vec{B}$
- (e) the angle between \vec{A} and \vec{B}
- (f) the area of the parallelogram with sides \vec{A}, \vec{B} .
- (g) write $\vec{A} = \vec{v}_1 + \vec{v}_2$ where \vec{v}_1 is parallel to \vec{B} and \vec{v}_2 is perpendicular to \vec{B} .

Problem 4 Let \vec{V} be a vector which makes an angle of 120° with the positive x-axis, 60° with the positive y-axis and 45° with the positive z-axis. In addition, you are given $\vec{V} \cdot \hat{x} = -10$. Find \vec{V} .

Problem 5 Suppose P, Q, R are the vertices of a triangle. Furthermore, suppose a, b are constants with a > 0 and P = (-1, 1, a) and Q = (1, b, 3) and R is at the origin. Also, you are given the interior angle at R is 90° . Finally, you are also given that P is a distance of $\sqrt{102}$ from R. Find a, b and find the angle interior to the triangle at P and Q.

Problem 6 Show that $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$.

Problem 7 Suppose \vec{A}, \vec{B} are perpendicular vectors. Use dot-products to show that

$$||\vec{A} + \vec{B}||^2 = ||\vec{A}||^2 + ||\vec{B}||^2.$$

Problem 8 Let $\vec{A} = \langle 2, 2, 1 \rangle$ and $\vec{B} = \langle 1, 1, -4 \rangle$. Find \vec{C} with length 1 which is orthogonal to both \vec{A} and \vec{B} . Then, find c_1, c_2, c_3 for which $\langle 3, 4, 5 \rangle = c_1 \vec{A} + c_2 \vec{B} + c_3 \vec{C}$.

Problem 9 Let a, b, c, d be constants. Find k for which $(a\vec{A} + b\vec{B}) \times (c\vec{A} + d\vec{B}) = k\vec{A} \times \vec{B}$.

Problem 10 Calculate $[\widehat{x} \times \langle a, b, c \rangle] \times \widehat{x}$ and $[\widehat{y} \times \langle a, b, c \rangle] \times \widehat{y}$. Conjecture the result of $[\widehat{z} \times \langle a, b, c \rangle] \times \widehat{z}$.

Problem 11 Let \hat{u} be a unit-vector. Let \vec{A} be an arbitrary vector. Show that:

$$\vec{A} = [\vec{A} \cdot \hat{u}]\hat{u} + [\hat{u} \times \vec{A}] \times \hat{u}.$$

Then, identify the given formulas with proj and orth operations as discussed in my notes (Defnition 1.1.24). The identity $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C}) \vec{B} - (\vec{A} \cdot \vec{B}) \vec{C}$ is helpful here.

Problem 12 Suppose $\vec{v} = \langle v_x, v_y \rangle$. Find the standard angle and magnitude of \vec{v} if

- (a) $v_x = 1 \text{ and } v_y = 2$
- (b) $v_x = -1 \text{ and } v_y = -2$
- (c) $v_x = 0 \text{ and } v_y = -3$

Problem 13 Suppose $\vec{r}(t) = \langle 2+3t, 3+t, 4-2t \rangle$ is the parametrization of a line which lies in a plane which also contains the point (1,4,8). Find the Cartesian equation of this plane. Finally, if a ninja is at (10,14,4) then how far is the ninja off the plane?

- **Problem 14** Suppose a line goes through P = (2, 3, 4) and Q = (3, 3, -3).
 - (a) find a parametrization $\vec{r}(t)$ of the line for which t=0 corresponds to (2,3,4)
 - (b) find a parametrization $\vec{R}(t)$ of the line for which t=0 corresponds to the midpoint of P and Q
 - (c) find the Cartesian equations of the line.
- **Problem 15** Consider the points P = (1,0,1), Q = (2,3,3) and R = (5,6,0). Consider the parallelogram with sides \overline{PQ} and \overline{PR} .
 - (a) find a parametrization $\vec{r}(s,t)$ of the parallelogram where $0 \le s,t \le 1$
 - (b) find the Cartesian equation of the plane which contains the parallelogram.
 - (c) write the parallelogram as a graph z = f(x, y) and be sure to explicitly find the appropriate domain for f.
 - (d) is the point (5/2, 9/2, 1/2) in the parallelogram?

Bonus: prove $\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$ using index calculation.