Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks!

Problem 21 Your signature below indicates you have:

(a.) I have read §1.5 – 1.6, 2.1 – 2.3 of Cook: ____________________.

(b.) I have attempted homeworks from Salas and Hille as listed below:________________.

The following homeworks from the text are good rudimentary skill problems. These are not collected or graded. However, they are all odd problems thus there are answers given within Salas, Hille and Etgen's text:

§ 9.3 #'s 21, 43, 61

§ 13.1 #'s 3, 5, 7, 15, 19, 25, 35, 43

§ 13.2 #'s 1,7,11, 17, 23, 29

§ 13.3 #'s 3,7,15

§ 13.4 #'s 5, 11, 13

§ 13.5 #'s 3, 11, 21

§ 13.7 #'s 7, 23, 27, 35

§ 14.1 #'s 3,7

§ 14.2 #'s 3,5,7,9,11,15,19,27,37

§ 14.3 #'s 3,5,11,17, 25, 29, 37,39,41

Problem 22 Find the parametric equations of the left half of a counterclockwise oriented circle of radius 4 which has center at $(2,2)$. Please include the domain of the parameter. Also, find the Cartesian equation of the whole circle.

Problem 23 Find the parametric equations of the curve of intersection of $ax + by + cz = d$ and the cylinder $y^2 + z^2 = R^2$. You may assume $a \neq 0$.

Problem 24 Consider the level surface $x^2 + 9y^2 = z^2$.

(a.) sketch the surface by hand and name the surface

(b.) find a parametrization of the surface.

Problem 25 Find the Cartesian equation of the surface with parametric equations:

$$x = \cosh\alpha \sin\beta, \qquad y = 2\cosh\alpha\cos\beta, \qquad z = \sinh\alpha.$$

Hint: $\cosh^2\alpha - \sinh^2\alpha = 1$.

Problem 26 Find the polar coordinate equations for

(a.) the ellipse $x^2/a^2 + y^2/b^2 = 1$,

(b.) the line $y = 1 - 2x$

Problem 27 Consider the point $P = (\sqrt{3}, 1, 2)$. Find the

(a.) cylindrical coordinates of P

(b.) spherical coordinates of P

Problem 28 Write the vector $\vec{v} = \langle 1, 2, 3 \rangle$ in terms of the spherical frame at an arbitrary point with cylindrical coordinates r, θ, z. In other words, find functions a, b, c of cylindrical coordinates such that $\vec{v} = a\widehat{r} + b\widehat{\theta} + c\widehat{z}$.
Hint: dot-products with respect to $\widehat{r}, \widehat{\theta}, \widehat{z}$ nicely isolate a, b and c if you make use of the fact that $\widehat{r}, \widehat{\theta}, \widehat{z}$ forms an orthonormal frame.

Problem 29 Find the intersection of $\phi = \pi/3$ and $z = 4$ and provide a parametrization which covers this curve of intersection.

Problem 30 Find the intersection of $\theta = \pi/4$ and $\rho = 4$ and provide a parametrization which covers this curve of intersection.

Problem 31 Let $\vec{A}(t) = \langle te^t,\ 2 + 3t,\ \sin(t^2) \rangle$ and calculate $\frac{d\vec{A}}{dt}$ and $\frac{d^2\vec{A}}{dt^2}$.

Problem 32 Suppose the velocity of Jeff is $\vec{v}(t) = \langle e^t, t, \sin t \rangle$ and Jeff is at the origin $(0, 0, 0)$ at $t = 0$. Find the acceleration, position and speed of Jeff at time t. Also, give the distance travelled during $[0, t]$ in terms of an integral.

Problem 33 Suppose a curve C is parametrized by $\vec{r}(t) = \langle 1 + t^2,\ \sqrt{t},\ \frac{1}{t+1} \rangle$. Find the parametrization of the tangent line to the given curve at $\vec{r}(2)$.

Problem 34 Show Theorem 2.1.16 part (1.) is true.

Problem 35 Let $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ give the position at time t. Find the tangential and normal components of the acceleration as described towards the conclusion of §2.3.

Problem 36 Find the tangent, normal and binormal vector fields for $\vec{\gamma}(t) = \langle 3t, 2\cos t, 2\sin t \rangle$.

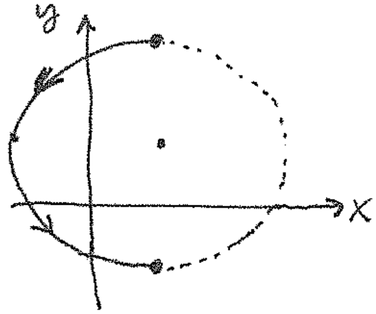
Problem 37 Reparametrize the curve given in the previous problem with respect to arclength.

Problem 38 Find the curvature and torsion for $\vec{\gamma}(t) = \langle 3t, 2\cos t, 2\sin t \rangle$.

Problem 39 The vector equation for the position $\vec{r}$ fixed on circle is given by $(\vec{r} - \vec{r}_o) \bullet (\vec{r} - \vec{r}_o) = R^2$ where R is a constant and $\vec{r}_o$ is the fixed center of the circle. Differentiate this equation directly and interpret your results in terms of centripetal and tangential acceleration.

Problem 40 Suppose $\frac{d\vec{A}}{dt} = \vec{A}$. Find the general solution. You should be able to solve this with separation of variables which is part of calculus II (a prerequisite of this course)

PROBLEM 22 find parametric eq$^{\underline{n}}$'s for <u>left</u> half of CCW, radius 4 circle centered at (2,2)



$$\left.\begin{aligned} x &= 2 + 4\cos t \\ y &= 2 + 4\sin t \\ \tfrac{\pi}{2} &\leq t \leq \tfrac{\pi}{2} \end{aligned}\right\} \quad \boxed{\begin{gathered}\vec{r}(t) = \langle 2+4\cos t,\ 2+4\sin t\rangle \\ \text{for } t \in [-\pi/2, \pi/2]\end{gathered}}$$

The Cartesian eq$^{\underline{n}}$ is $(x-2)^2 + (y-2)^2 = 16$.

PROBLEM 23 Find parametric eq$^{\underline{n}}$'s of $ax+by+cz=d$ and $y^2+z^2=R^2$ curve of <u>intersection</u>. Assume $a \neq 0$ (and $R \neq 0$)

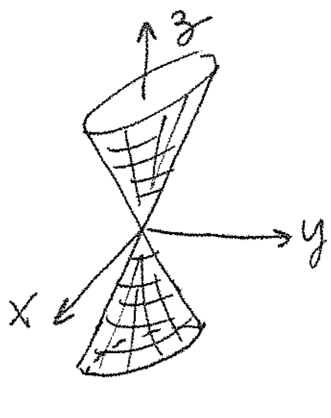
Let $\underset{*}{\underline{y = R\cos\theta}}$ and $\underset{*}{\underline{z = R\sin\theta}}$ then $y^2+z^2 = R^2(\cos^2\theta + \sin^2\theta) = R^2$ hence the curve with $\underset{*}{\underline{\text{such}}}$ will be on cylinder. To put the curve on the plane we need x to solve:

$$ax + bR\cos\theta + cR\sin\theta = d$$
$$\hookrightarrow x = \frac{d - bR\cos\theta - cR\sin\theta}{a}$$

Thus,
$$\boxed{x = \frac{d - bR\cos\theta - cR\sin\theta}{a},\quad y = R\cos\theta,\quad z = R\sin\theta}$$
parametrize the curve of intersection..

PROBLEM 24 Consider $S:\ x^2+9y^2=z^2$ < (a.) sketch & name the surface; (b.) parametrize surface

(a.) this is an elliptical cone. If we fix $z = z_0$ then $\dfrac{x^2}{z_0^2} + \dfrac{y^2}{(z_0/3)^2} = 1$ so each horizontal slice reveals an ellipse.



(b.) Many choices possible, my preferred choice θ, z

$$\boxed{\vec{r}(\theta,z) = \left\langle z\cos\theta,\ \frac{z}{3}\sin\theta,\ z\right\rangle} \quad \begin{gathered} 0 \leq \theta \leq 2\pi \\ z \in \mathbb{R}. \end{gathered}$$

If you set $x = z\cos\theta$ and $y = \dfrac{z\sin\theta}{3}$ you can easily see $x^2 + 9y^2 = z^2$.

PROBLEM 25 Suppose a surface S has parametrization given by $x = \cosh\alpha \sin\beta$, $y = 2\cosh\alpha\cos\beta$, $z = \sinh\alpha$. Find the cartesian eq^n to describe S.

Note $x^2 + \left(\frac{y}{2}\right)^2 = \cosh^2\alpha \sin^2\beta + \cosh^2\alpha\cos^2\beta$
$= \cosh^2\alpha\,[\sin^2\beta + \cos^2\beta]$
$= \cosh^2\alpha$

Hence $x^2 + \left(\frac{y}{2}\right)^2 - z^2 = \cosh^2\alpha - \sinh^2\alpha = 1 \quad \therefore \boxed{x^2 + \frac{y^2}{4} - z^2 = 1}$

this is a hyperboloid of one sheet (not that I asked for a name here)

PROBLEM 26 Find polar eq^n's for a.) $x^2/a^2 + y^2/b^2 = 1$, b.) $y = 1 - 2x$

This problem is simply done by setting $x = r\cos\theta$, $y = r\sin\theta$,

(a.) $\underbrace{\frac{(r\cos\theta)^2}{a^2} + \frac{(r\sin\theta)^2}{b^2} = 1}$ $\longleftrightarrow$ $r^2\left[\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}\right] = 1$

also a good answer but later these will be useful.

$$\boxed{r = \frac{1}{\sqrt{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}}}}$$

(b.) $r\sin\theta = 1 - 2\cos\theta$

$$\boxed{r = \frac{1}{\sin\theta + 2\cos\theta}}$$

PROBLEM 27 Consider $P = (\sqrt{3}, 1, 2)$ find the cylindrical (a.) and spherical (b.) coordinates for P.

(a.) $r = \sqrt{3+1} = \sqrt{4} = 2.$
$\theta = \tan^{-1}(1/\sqrt{3}) = \tan^{-1}\left(\frac{1/2}{\sqrt{3}/2}\right) = \pi/6 \quad \Rightarrow \boxed{(r, \theta, z)_{cyl.} = (2, \frac{\pi}{6}, 2)}$

(b.) $\rho = \sqrt{3+1+4} = \sqrt{8}$
$\theta = \pi/6$
$\underbrace{\phi = \cos^{-1}\left(\frac{2}{\sqrt{8}}\right)} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pi/4 \quad \therefore \boxed{(\rho, \phi, \pi) = (\sqrt{8}, \frac{\pi}{4}, \frac{\pi}{6})}$

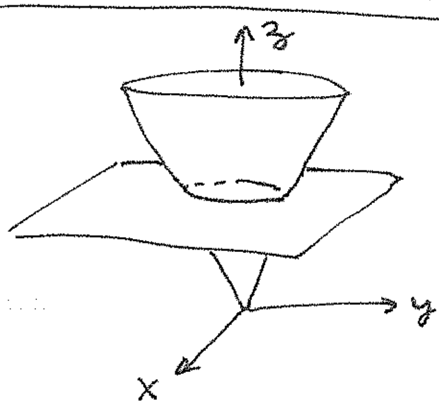
$z = \rho\cos\phi$
$\phi = \cos^{-1}[z/\rho]$

PROBLEM 28 Let $\vec{v} = \langle 1, 2, 3\rangle$ find a, b, c s.t. $\vec{v} = a\hat{r} + b\hat{\theta} + c\hat{z}$

$a = \vec{v}\cdot\hat{r} = \langle 1,2,3\rangle \cdot \langle \cos\theta, \sin\theta, 0\rangle = \cos\theta + 2\sin\theta$
$b = \vec{v}\cdot\hat{\theta} = \langle 1,2,3\rangle \cdot \langle -\sin\theta, \cos\theta, 0\rangle = -\sin\theta + 2\cos\theta$
$c = \vec{v}\cdot\hat{z} = \langle 1,2,3\rangle \cdot \langle 0,0,1\rangle = 3$

$$\therefore \boxed{\vec{v} = (\cos\theta + 2\sin\theta)\hat{r} + (2\cos\theta - \sin\theta)\hat{\theta} + 3\hat{z}}$$

PROBLEM 29 Find intersection of $\phi = \pi/3$, $z = 4$ and provide parametrization of this curve.



$x = \rho \cos\theta \sin \pi/3$

$y = \rho \sin\theta \sin \pi/3$

$z = \rho \cos \pi/3 = \rho/2 = 4 \Rightarrow \rho = 8$ for the curve

Hence, $x = \frac{8\sqrt{3}}{2}\cos\theta = 4\sqrt{3}\cos\theta$

and $y = 4\sqrt{3}\sin\theta$ while $z = 4$

We find θ is natural parameter and $\boxed{\vec{r}(\theta) = \langle 4\sqrt{3}\cos\theta, 4\sqrt{3}\sin\theta, 4\rangle}$

PROBLEM 30 Find parametrization of curve of intersection of $\theta = \pi/4$ and $\rho = 4$

$$\left.\begin{aligned} x &= \rho\cos\theta\sin\phi = 4\cos\tfrac{\pi}{4}\sin\phi \\ y &= \rho\sin\theta\sin\phi = 4\sin\tfrac{\pi}{4}\sin\phi \\ z &= \rho\cos\phi = 4\cos\phi \end{aligned}\right\} \quad \boxed{\vec{r}(\phi) = \langle \sqrt{8}\sin\phi, \sqrt{8}\sin\phi, 4\cos\phi\rangle}$$

$4/\sqrt{2} = \sqrt{\frac{16}{2}}$ $\qquad 0 \leq \phi \leq \pi$.

(this is a half-circle formed from intersection of half-plane $\theta = \pi/4$ and the sphere $\rho = 4$.)

PROBLEM 31 Let $\vec{A}(t) = \langle te^t, 2+3t, \sin(t^2)\rangle$, find $\vec{A}'$ & $\vec{A}''$

$$\frac{d\vec{A}}{dt} = \left\langle \frac{d}{dt}(te^t), \frac{d}{dt}(2+3t), \frac{d}{dt}(\sin(t^2))\right\rangle = \langle e^t + te^t, 3, 2t\cos(t^2)\rangle$$

$$\boxed{\frac{d\vec{A}}{dt} = \langle e^t(1+t), 3, 2t\cos(t^2)\rangle}$$

$$\frac{d^2\vec{A}}{dt^2} = \frac{d}{dt}\left(\frac{d\vec{A}}{dt}\right) = \langle e^t + e^t(1+t), 0, 2(\cos(t^2) - 2t^2\sin(t^2))\rangle$$

$$\boxed{\frac{d^2\vec{A}}{dt^2} = \langle e^t(2+t), 0, 2\cos(t^2) - 4t^2\sin(t^2)\rangle}$$

$\boxed{\text{PROBLEM 32}}$ Given velocity $\vec{V}(t) = \langle e^t, t, \sin t\rangle$ and $\vec{r}(0) = \langle 0,0,0\rangle$ find $\underbrace{\vec{a} = \frac{d\vec{V}}{dt}}_{\text{acceleration}}$ and $\underbrace{\vec{r}(t)}_{\text{position}}$ and $\underbrace{V(t)}_{\text{speed}}$. Finally, provide integral formulas for distance travelled from 0 to t.

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}\langle e^t, t, \sin t\rangle = \boxed{\langle e^t, 1, \cos t\rangle = \vec{a}(t)}$$

$$V = \|\vec{V}\| = \sqrt{(e^t)^2 + t^2 + \sin^2 t} \ \therefore \ \boxed{V(t) = \sqrt{e^{2t} + t^2 + \sin^2 t}}$$

Note $\vec{V} = \frac{d\vec{r}}{dt} \Rightarrow \vec{r}(t) = \int \vec{V}(t)\,dt = \langle e^t, \frac{1}{2}t^2, -\cos t\rangle + \vec{C}$

However, $\vec{r}(0) = \langle 1, 0, -1\rangle + \vec{C} = \langle 0,0,0\rangle \Rightarrow \vec{C} = \langle -1, 0, 1\rangle$

So ~~we~~ find position $\boxed{\vec{r}(t) = \langle e^t - 1, t^2/2, 1-\cos t\rangle}$

To find distance travelled we integrate $\frac{ds}{dt}$ w.r.t. to t to find net-arclength of the path, Note, $V = \frac{ds}{dt}$

$$\boxed{\text{distance travelled} = \int_0^t \underbrace{\sqrt{e^{2u} + u^2 + \sin^2 u}\, du}_{\text{shouldn't use } t \text{ here as it already has a set-meaning given by problem statement.}}}$$

$\boxed{\text{PROBLEM 33}}$

$\boxed{C: \vec{r}(t) = \langle 1+t^2, \sqrt{t}, \frac{1}{t+1}\rangle. \text{ Find tangent line to } C \text{ at } \vec{r}(2)}$

The tangent line is parametrized by $\vec{l}(t) = \vec{r}(2) + t\vec{V}(2)$ or, if we want t to line-up with $\vec{r}(t)$ use $\vec{l}(t) = \vec{r}(2) + (t-2)\vec{V}(2)$.

$$\frac{d\vec{r}}{dt} = \langle 2t, \frac{1}{2\sqrt{t}}, \frac{-1}{(t+1)^2}\rangle$$

$$\vec{V}(2) = \langle 4, \frac{1}{2\sqrt{2}}, \frac{-1}{9}\rangle \quad , \vec{r}(2) = \langle 5, \sqrt{2}, \frac{1}{3}\rangle$$

Thus

$$\boxed{\vec{l}(t) = \langle 5, \sqrt{2}, \frac{1}{3}\rangle + t\langle 4, \frac{1}{2\sqrt{2}}, \frac{-1}{9}\rangle}$$

(other answers possible, one nice one,

$$\vec{l}_2(t) = \langle 5, \sqrt{2}, 1/3\rangle + (t-2)\langle 4, \frac{1}{2\sqrt{2}}, \frac{-1}{9}\rangle$$

PROBLEM 34 Show Th^{m} 2.1.16 part (1.) is true

Let $\vec{A}, \vec{B} : J \subseteq \mathbb{R} \longrightarrow \mathbb{R}^n$ be differentiable then

$$\begin{aligned}
\frac{d}{dt}\left(\vec{A}+\vec{B}\right) &= \frac{d}{dt}\langle A_1+B_1, A_2+B_2, \dots, A_n+B_n\rangle \\
&= \left\langle \frac{d}{dt}(A_1+B_1), \frac{d}{dt}(A_2+B_2), \dots, \frac{d}{dt}(A_n+B_n)\right\rangle \\
&= \left\langle \frac{dA_1}{dt}+\frac{dB_1}{dt}, \frac{dA_2}{dt}+\frac{dB_2}{dt}, \dots, \frac{dA_n}{dt}+\frac{dB_n}{dt}\right\rangle \\
&= \left\langle \frac{dA_1}{dt}, \frac{dA_2}{dt}, \dots, \frac{dA_n}{dt}\right\rangle + \left\langle \frac{dB_1}{dt}, \frac{dB_2}{dt}, \dots, \frac{dB_n}{dt}\right\rangle \\
&= \frac{d\vec{A}}{dt} + \frac{d\vec{B}}{dt} \quad //
\end{aligned}$$

Alternatively,

$$\begin{aligned}
\left[\frac{d}{dt}\left(\vec{A}+\vec{B}\right)\right]_i &= \frac{d}{dt}\left(\vec{A}+\vec{B}\right)_i && : \text{def}^{\underline{n}} \text{ of derivative of vector-valued function.} \\
&= \frac{d}{dt}\left(A_i+B_i\right) && : \text{def}^{\underline{n}} \text{ of vector addition.} \\
&= \frac{dA_i}{dt}+\frac{dB_i}{dt} && : \text{ordinary derivative property.} \\
&= \left(\frac{d\vec{A}}{dt}\right)_i + \left(\frac{d\vec{B}}{dt}\right)_i && : \text{def}^{\underline{n}} \text{ of derivative of } \vec{A} \text{ \& } \vec{B}. \\
&= \left(\frac{d\vec{A}}{dt}+\frac{d\vec{B}}{dt}\right)_i && : \text{def}^{\underline{n}} \text{ of vector add. once again.}
\end{aligned}$$

Hence, as the above holds for $i = 1, 2, \dots, n$ we derive $\frac{d}{dt}\left(\vec{A}+\vec{B}\right) = \frac{d\vec{A}}{dt}+\frac{d\vec{B}}{dt}$ //

PROBLEM 35 Find a_N and a_T such that $\vec{a} = a_N \hat{N} + a_T \hat{T}$ where $\vec{r}(t) = \langle t, t^2, t^3 \rangle$ (Note, $N \cdot T = 0$ so $a_T = \vec{a} \cdot T$ & $a_N = \vec{a} \cdot N$.)

$$\vec{v}(t) = \langle 1, 2t, 3t^2 \rangle \implies T(t) = \frac{1}{\sqrt{1+4t^2+9t^4}} \langle 1, 2t, 3t^2 \rangle$$

$$\vec{a}(t) = \langle 0, 2, 6t \rangle \text{ thus } a_T = \vec{a} \cdot T = \boxed{\frac{4t + 18t^3}{\sqrt{1+4t^2+9t^4}} = a_T}$$

Calculation of N is a pain,

$$\frac{dT}{dt} = \frac{d}{dt}\left[\frac{1}{\sqrt{1+4t^2+9t^4}} \langle 1, 2t, 3t^2 \rangle\right] \quad \left\{\frac{d}{dt}(f\vec{A}) = \frac{df}{dt}\vec{A} + f\frac{d\vec{A}}{dt}\right\}$$

$$= \frac{8t + 36t^3}{-2(1+4t^2+9t^4)^{3/2}} \langle 1, 2t, 3t^2 \rangle + \frac{1}{(1+4t^2+9t^4)^{1/2}} \langle 0, 2, 6t \rangle$$

$$= \frac{1}{\sqrt{1+4t^2+9t^4}}\left[-\frac{4t+18t^3}{1+4t^2+9t^4}\langle 1, 2t, 3t^2 \rangle + \langle 0, 2, 6t \rangle\right]$$

$$= \frac{1}{\sqrt{1+4t^2+9t^4}}\left[\left\langle \frac{-4t-18t^3}{1+4t^2+9t^4}, \frac{-8t^2-36t^4}{1+4t^2+9t^4} + 2, \frac{-12t^3-54t^5}{1+4t^2+9t^4} + 6t \right\rangle\right]$$

$$= \frac{1}{\sqrt{1+4t^2+9t^4}}\left[\left\langle \frac{-4t-18t^3}{1+4t^2+9t^4}, \frac{\cancel{-8t^2}-36t^4+2+\cancel{8t^2}+18t^4}{1+4t^2+9t^4}, \frac{-12t^3-\cancel{54t^5}+6t+24t^3+\cancel{54t^5}}{1+4t^2+9t^4} \right\rangle\right]$$

$$= \frac{1}{(1+4t^2+9t^4)^{3/2}} \langle -4t-18t^3, -18t^4+2, 12t^3+6t \rangle$$

$$= \underbrace{\frac{2}{(1+4t^2+9t^4)^{3/2}} \langle -2t-9t^3, 1-9t^4, 3t+6t^3 \rangle}$$

N is this vector normalized. Now is the time I was warning of, ignore the coeff. fact and just normalize the vector!

$$N = \frac{1}{\sqrt{(9t^3+2t)^2 + (1-9t^4)^2 + (3t+6t^3)^2}} \langle -2t-9t^3, 1-9t^4, 3t+6t^3 \rangle$$

Thus, as $a_N = N \cdot \vec{a}$,

$$\boxed{a_N = \frac{2 - 18t^4 + 6t(3t+6t^3)}{\sqrt{(9t^3+2t)^2 + (1-9t^4)^2 + (3t+6t^3)^2}}}$$

this could be simplified, we could also check that $\vec{a} = a_T T + a_N N$ (it is likely I've made a mistake by now ☺ see

PROBLEM 35 continued

Since we know $\vec{a} = a_T T + a_N N$ we can find $a_N N$ without calculating N (which was a pain here!)

$$
\begin{aligned}
a_N N &= \vec{a} - a_T T \\
&= \langle 0, 2, 6t \rangle - \frac{4t + 18t^3}{\sqrt{1+4t^2+9t^4}} \frac{\langle 1, 2t, 3t^2 \rangle}{\sqrt{1+4t^2+9t^4}} \\
&= \langle 0, 2, 6t \rangle - \frac{4t + 18t^3}{1+4t^2+9t^4} \langle 1, 2t, 3t^2 \rangle \\
&= \left\langle \frac{-4t - 18t^3}{1+4t^2+9t^4}, \ 2 + \frac{-8t^2 - 36t^4}{1+4t^2+9t^4}, \ 6t + \frac{-12t^3 - 54t^5}{1+4t^2+9t^4} \right\rangle \\
&= \left\langle \frac{-4t - 18t^3}{1+4t^2+9t^4}, \ \frac{2 + \cancel{8t^2} + 18t^4 - \cancel{8t^2} - 36t^4}{1+4t^2+9t^4}, \ \frac{6t + 24t^3 + \cancel{54t^5} - 12t^3 - \cancel{54t^5}}{1+4t^2+9t^4} \right\rangle \\
&= \frac{1}{1+4t^2+9t^4} \langle -4t - 18t^3, \ 2 - 18t^4, \ 6t + 12t^3 \rangle
\end{aligned}
$$

Hence,

$$
a_N = \| a_N N \| = \underline{\frac{1}{1+4t^2+9t^4} \sqrt{(4t+18t^3)^2 + (2-18t^4)^2 + (6t+12t^3)^2}}
$$

Oh, so, now you know why I told you, just find a_T.

PROBLEM 36

$$\vec{\gamma}(t) = \langle 3t,\ 2\cos t,\ 2\sin t \rangle$$

$$\vec{\gamma}'(t) = \langle 3,\ -2\sin t,\ 2\cos t \rangle \hookrightarrow \|\vec{\gamma}'(t)\| = \sqrt{9+4\sin^2 t+4\cos^2 t}$$

$$\|\vec{\gamma}'(t)\| = \sqrt{13}.$$

$$\therefore\ \boxed{T(t) = \frac{1}{\sqrt{13}}\langle 3,\ -2\sin t,\ 2\cos t \rangle}\ \text{(tangent)}$$

$$T'(t) = \frac{1}{\sqrt{13}}\langle 0,\ -2\cos t,\ -2\sin t \rangle \Rightarrow \boxed{N(t) = \langle 0, -\cos t, -\sin t \rangle}$$

(normal)

The binormal is defined by $B = T \times N$.

$$B(t) = T(t) \times N(t)$$

$$= \left\langle \frac{3}{\sqrt{13}},\ \frac{-2\sin t}{\sqrt{13}},\ \frac{2\cos t}{\sqrt{13}} \right\rangle \times \langle 0, -\cos t, -\sin t \rangle$$

$$= \left\langle \frac{2}{\sqrt{13}}(\sin^2 t+\cos^2 t),\ 0+\frac{3}{\sqrt{13}}\sin t,\ \frac{-3}{\sqrt{13}}\cos t \right\rangle$$

$$= \boxed{\frac{1}{\sqrt{13}}\langle 2,\ 3\sin t,\ -3\cos t \rangle = B(t)}\ \text{(binormal)}$$

As a check, $B \cdot N = 0$ and $B \cdot T = 0$.

PROBLEM 37

Arclength $s(t) = \int_0^t \|\vec{\gamma}'(\bar{t})\|\, d\bar{t} = \int_0^t \sqrt{13}\, d\bar{t} = t\sqrt{13}$

thus $t = s/\sqrt{13}$ hence

$$\boxed{\tilde{\gamma}(s) = \left\langle \frac{3s}{\sqrt{13}},\ -2\sin\left(\frac{s}{\sqrt{13}}\right),\ 2\cos\left(\frac{s}{\sqrt{13}}\right) \right\rangle}$$

$\boxed{\text{PROBLEM 38}}$ Calculate curvature and torsion for $\vec{\gamma}(t) = \langle 3t, 2\cos t, 2\sin t \rangle$

The Frenet-Serret Eq^{n}'s for arbitrary speed v are

$$\frac{dT}{dt} = \kappa v N \quad , \quad \frac{dN}{dt} = -\kappa v T + \tau v B \, , \quad \frac{dB}{dt} = -\tau v N$$

these go hand-in-hand with the definitions,

$$\boxed{\kappa = \frac{1}{v}\frac{dT}{dt} \cdot N \quad \& \quad \tau = \frac{-1}{v}\frac{dB}{dt} \cdot N}$$

there are other techniques to calculate κ & τ but I'll use the boxed def^{n} as our guide here. We already found T, N, B in $\boxed{\text{PROBLEM 36}}$

$$T(t) = \frac{1}{\sqrt{13}} \langle 3, -2\sin t, 2\cos t \rangle \qquad (v = \sqrt{13})$$

$$N(t) = \langle 0, -\cos t, -\sin t \rangle$$

$$B(t) = \frac{1}{\sqrt{13}} \langle 2, 3\sin t, -3\cos t \rangle$$

Thus calculate,

$$\frac{dT}{dt} = \frac{1}{\sqrt{13}} \langle 0, -2\cos t, -2\sin t \rangle$$

$$\Rightarrow \frac{dT}{dt} \cdot N = \frac{1}{\sqrt{13}} \langle 0, -2\cos t, -2\sin t \rangle \cdot \langle 0, -\cos t, -\sin t \rangle = \frac{2}{\sqrt{13}}$$

$$\therefore \kappa = \frac{1}{\sqrt{13}}\left(\frac{2}{\sqrt{13}}\right) = \frac{2}{13} \qquad \therefore \boxed{\kappa = 2/13}$$

Likewise,

$$\frac{dB}{dt} \cdot N = \frac{1}{\sqrt{13}} \langle 0, 3\cos t, 3\sin t \rangle \cdot \langle 0, -\cos t, -\sin t \rangle = \frac{-3}{\sqrt{13}}$$

$$\Rightarrow \tau = \frac{-1}{\sqrt{13}}\left(\frac{-3}{\sqrt{13}}\right) \Rightarrow \boxed{\tau = 3/13}$$

PROBLEM 39 Given $(\vec{r}-\vec{r}_0)\cdot(\vec{r}-\vec{r}_0) = R^2$ describes circular motion, differentiate (w.r.t. time t) and interpret w.r.t centripetal & tangential acceleration.

Diff. w.r.t. t, note $\frac{d}{dt}(R^2) = 0$ for the zero on the LHS below,

$$0 = \frac{d}{dt}\left(\left(\vec{r}-\vec{r}_0\right)\cdot\left(\vec{r}-\vec{r}_0\right)\right) = \frac{d}{dt}\left(\vec{r}-\vec{r}_0\right)\cdot\left(\vec{r}-\vec{r}_0\right) + \left(\vec{r}-\vec{r}_0\right)\cdot\frac{d}{dt}\left(\vec{r}-\vec{r}_0\right)$$

$$= \frac{d\vec{r}}{dt}\cdot\left(\vec{r}-\vec{r}_0\right) + \left(\vec{r}-\vec{r}_0\right)\cdot\frac{d\vec{r}}{dt}$$

$$= 2\underbrace{\left(\vec{r}-\vec{r}_0\right)}\cdot\underbrace{\frac{d\vec{r}}{dt}}$$

vector from center of circle $\vec{r}_0$ to point $\vec{r}$ on circle

velocity or geometrically direction of tangent line.

- the eq$^{\underline{n}}$ above shows velocity is $\perp$ to radius of circle for circular motion.

Continuing, diff. $0 = \left(\vec{r}-\vec{r}_0\right)\cdot\vec{v}$ w.r.t. t,

$$0 = \frac{d}{dt}\left[\left(\vec{r}-\vec{r}_0\right)\cdot\vec{v}\right] = \frac{d}{dt}\left(\vec{r}-\vec{r}_0\right)\cdot\vec{v} + \left(\vec{r}-\vec{r}_0\right)\cdot\frac{d\vec{v}}{dt}$$

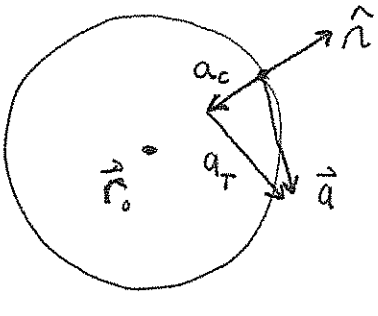
$$\Rightarrow \quad \frac{d\vec{r}}{dt}\cdot\vec{v} + \left(\vec{r}-\vec{r}_0\right)\cdot\vec{a} = 0$$

$$\Rightarrow \quad \vec{v}\cdot\vec{v} = -\left(r-\vec{r}_0\right)\cdot\vec{a}$$

$$\Rightarrow \quad v^2 = -\left(\Omega\hat{\Omega}\right)\cdot\vec{a}$$

$$\Rightarrow \quad \vec{a}\cdot\hat{\Omega} = \frac{-v^2}{\Omega} \Rightarrow \boxed{a_c = \frac{-v^2}{\Omega}}$$

See Example 2.3.2 of the Lecture Notes for another approach



By geometry $\hat{\Omega} = -N$ and as $\vec{a} = a_T T + a_N N$ we have

$$a_T T = \vec{a} - a_N N$$

$$\Rightarrow \quad a_T T = \vec{a} + \frac{v^2}{\Omega} N$$

$\boxed{\text{PROBLEM 40}}$ Suppose $\frac{d\vec{A}}{dt} = \vec{A}$. I'll suppose $\vec{A} = \langle a, b \rangle$ but we can easily generalize to $\vec{A} = \langle A_1, A_2, .., A_n \rangle$.

$$\frac{d\vec{A}}{dt} = \vec{A} \Rightarrow \left\langle \frac{da}{dt}, \frac{db}{dt} \right\rangle = \langle a, b \rangle$$

$$\Rightarrow \frac{da}{dt} = a \quad \& \quad \frac{db}{dt} = b$$

$$\Rightarrow \frac{da}{a} = dt \quad \& \quad \frac{db}{b} = dt$$

$$\Rightarrow \ln|a| = t + c_1 \quad \& \quad \ln|b| = t + c_2$$

$$\Rightarrow |a| = e^{t+c_1} \quad \& \quad |b| = e^{t+c_2}$$

$$\Rightarrow a = \pm e^{c_1} e^t \quad \& \quad b = \pm e^{c_2} e^t$$

$$\Rightarrow a = k_1 e^t \quad \& \quad b = k_2 e^t$$

$$\therefore \boxed{\vec{A}(t) = \langle k_1, k_2 \rangle e^t} \quad (\text{note, } \vec{A}(0) = \langle k_1, k_2 \rangle).$$

Likewise, for $\mathbb{R}^n$, $\vec{A}(t) = e^t \vec{A}_0$ where $\vec{A}(0) = \vec{A}_0$.