Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks!

Problem 41 Your signature below indicates you have:

(a.) I have read Chapter 3 and §4.1 − 4.3 of Cook: ____________________.

(b.) I have attempted homeworks from Salas and Hille as listed below:_________________.

The following homeworks from the text are good rudimentary skill problems. These are not collected or graded. However, they are all odd problems thus there are answers given within Salas, Hille and Etgen's text:

§ 14.1 #'s 1, 5, 11, 27

§ 14.4 #'s 1, 5, 9, 13, 15, 17, 21, 27, 31, 33, 47

§ 14.5 #'s 1, 3, 5, 9

§ 14.6 #'s 1, 5, 11, 15, 17, 21, 23, 27

§ 15.1 #'s 1, 7, 15, 17, 25, 29

§ 15.2 #'s 1, 5, 11, 19, 25, 33, 37

Problem 42 Prove by a picture that the half-plane $H = \{(x, y) \mid y > 0\}$ is indeed open. Your picture should illustrate why each point in H is an interior point.

Problem 43 Consider the annulus $A = \{(x, y) \mid 1 \leq x^2 + y^2 < 4\}$. Find the boundary of A. Again, include a sketch of every disk centered on the claimed boundary point has points both inside and outside the set.

Problem 44 Let $f(x, y) = \frac{\sin(x^2+y^2)}{x^2+y^2}$ for $(x, y) \neq 0$ and $f(0, 0) = A$. Choose the appropriate value for A such that f is continuous on $\mathbb{R}^2$ and show, relative to that choice, f is indeed continuous on $\mathbb{R}^2$.

Problem 45 Let $f(x, y) = \frac{x^2 y}{x^2+y^2}$ for $(x, y) \neq 0$ and $f(0, 0) = A$. If possible, choose the appropriate value for A such that f is continuous on $\mathbb{R}^2$ and show, relative to that choice, f is indeed continuous on $\mathbb{R}^2$. Or, if no choice of A makes f continuous at $(0, 0)$ explain why by explicitly showing the limit at $(0, 0)$ of f does not exist.

Problem 46 Calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for:

(a) Let $f(x, y) = e^x \sin(xy)$.

(b) Let $f(x, y) = \ln\left(e^{x+y}\sqrt{x^2 + y^2}\right)$.

Problem 47 Suppose $g(x,y) = e^{x-y}$. Evaluate $g, g_x, g_y, g_{xx}, g_{xy}, g_{yy}$ at $(0,0)$. What can you say about the values as they compare to the series formed by $e^u = 1+u+\frac{1}{2}u^2+\cdots$ with $u = x^2-y^2$. Use this example to propose[1] the second order Taylor polynomial centered at $(0,0)$.

Problem 48 Let $f(\rho,\theta,\phi) = \tan^2(\rho+\phi\theta^2)$. Calculate $\partial_\theta\partial_\rho\partial_\phi f$.

Problem 49 Show that $u = e^x\cos(y)$ and $v = e^x\sin(y)$ are solutions of $\Phi_{xx}+\Phi_{yy} = 0$. This equation is known as Laplace's Equation and it can also be written as $\nabla^2\Phi = 0$.

Problem 50 Suppose $f(x_1,x_2,\dots,x_n) = \sqrt{x_1^2+x_2^2+\cdots+x_n^2}$. Calculate $\frac{\partial f}{\partial x_1}$

Instructions for following two problems: Calculate the total differential and gradient for the functions given below. The total differential of $f(x_1,x_2,\dots,x_n)$ is

$$df = (\partial_1 f)dx_1 + (\partial_2 f)dx_2 + \cdots + (\partial_n f)dx_n$$

and the gradient of $f(x_1,x_2,\dots,x_n)$ is $\nabla f = \langle \partial_1 f, \partial_2 f, \dots, \partial_n f\rangle$.

Problem 51 (a) $f(x,y) = \sqrt{x^2+y^2}$

(b) $g(x,y) = [f(x,y)]^2 = x^2+y^2$

Problem 52 $h(x_1,x_2,\dots,x_n) = x_1^2+x_2^2+\cdots x_n^2$.
(notice $h = f^2$ where f is the function studied in Problem 50 , note the similarity with the preceding problem)

Problem 53 Let $f(x,y) = 3+xy+x^3$. Consider the point $p = (2,1)$. Find unit-vector(s) based at p which point in the direction in which f changes at the rate of 12 if possible. Also, if possible, find the unit-vectors which point in the direction in which f changes at rate 15 at p.

Problem 54 Let $f(x,y) = e^{yx^2}$. Find the rate of change of f at $(0,0)$ in the $\langle a,b\rangle$-direction. If L is a line through the origin which is perpendicular to the directions in which the maximum and minimum rates of change of f at $(0,0)$ occur then what special property do we observe for $f(x,y)$ where $(x,y)\in L$?

Problem 55 Calculate ∇f for each of the functions below:

(a) $f(x,y) = 2x+3y$

(b) $f(x,y) = exp(-x^2+2x-y^2)$

(c) $f(x,y) = \sin(x+y)$

Problem 56 What is the rate of change for each of the functions given in the previous problem at the point $(1,3)$ in the direction of the vector $\langle 1,-1\rangle$.

Problem 57 Again, concerning the functions given in Problem 62, in what directions are the functions locally constant at the point $(1,3)$? (give answers in terms of unit-direction vectors)?

[1]spolier alert; You can check your claim against page 254 of my notes.

Problem 58 Suppose the temperature T is a function of the coordinates x, y in a large plane of battle. Furthermore, suppose the enemy ninja is carefully building a large attack by molding chakra over some time. During the preparation of the attack the enemy is vulnerable to your attack. Knowing this he has obscured your field of vision with multiple smoke bombs. However, the mass of energy building actually heats the ground. Fortunately one of your ninja skills is temperature sensitivity. You extrapolate from the temperature of the ground near your location that the temperature function has the form $T(x, y) = 50 + 2x + 3y$. In what direction should you attack?

Problem 59 Suppose $\vec{F}(x, y, z) = \langle 2xy^2, 2x^2y, 3 \rangle$. What scalar function f yields $\vec{F}$ as a gradient vector field? Find f such that $\nabla f = \vec{F}$.
(here we have to work backwards, write down what you want and guess, by the way, the function $-f$ is the pontial energy function for the force field $\vec{F}$.)

Problem 60 Consider x, y the cartesian coordinate functions on $\mathbb{R}^2$. Prove from the limit-based definition of partial derivatives that $\frac{\partial y}{\partial x} = 0$ and $\frac{\partial x}{\partial x} = 0$.

Bonus Problem: [use of technology to solve algebraic and/or transcendental equation that the problem suggests] The temperature in an air conditioned room is set at 65. A ninja with expert ocular jutsu disguises himself in plain sight by bending light near him with his art. However, his art does not extend to the infared spectrum and his body heat leaves a signature variation in the otherwise constant room temperature. In particular,

$$T(x, y, z) = 33 \exp\left[\frac{-(x-3)^2 - (y-4)^2 - (z-1)^2}{10}\right] + 65.$$

Shino searches for the cloaked ninja by sending insect scouts which are capable of sensing a change in temperature as minute as 0.1 degree per meter. How close do the scout insects have to get before they sense the hidden ninja? (also, where is the hidden ninja and what is his body temperature on the basis of the given T which is in meters and degrees Farenheight)