Same instructions as in Mission 1. See Mission 1 for details.

Problem 31 Your PRINTED NAME below indicates you have:

(a.) Your PRINTED NAME below indicates you have:

(a.) I have read Chapter 3 and §4.1 – 4.3 of Cook: ____________________.

(b.) I have attempted 10 problems either from Stewart(see below) or the end of chapter problems in my notes:________________.

§ 14.1 #'s 9, 11, 13, 15, 17, 20, 23, 29, 32, 45, 48, 61, 63, 65, 67, 68, 69, 70

§ 14.2 #'s 5, 7, 9, 11, 13, 15, 17, 19, 21, 29, 31, 37, 39, 41, 42

§ 14.3 #'s 15, 19, 23, 25, 27, 29, 31, 39, 41, 43, 47, 49, 53, 57, 61, 63, 67, 71, 76, 77, 88

Problem 32 Consider the set $S = \{(x, y) \mid x^2 + y^2 < 1 \ \& \ y \geq 0\}$. Sketch the set and picture an open disk around a typical interior point as well as an open disk around a typical boundary point. Which part of the boundary is not in S ?

Problem 33 Let $f(x, y) = \sqrt{x^2 - y^2}$. Find the domain of f and determine where f is continuous. Let $f(x, y) = \frac{\sin(x^2+y^2)}{x^2+y^2}$ for $(x, y) \neq 0$ and $f(0, 0) = A$. Choose the appropriate value for A such that f is continuous on $\mathbb{R}^2$ and show, relative to that choice, f is indeed continuous on $\mathbb{R}^2$

Problem 34 Determine the value of the limit at (a, b) for each point in the plane. If the limit does not exist at a particular choice then supply a proper argument to demonstrate the non-existence of that limit.

$$\lim_{(x,y)\to(a,b)} \frac{(x-1)^2y^2}{(x^3+y^3)(x-1)}.$$

Problem 35 Calculate $\nabla f = \langle f_x, f_y \rangle$ for each of the functions below:

(a) $f(x, y) = \ln\left(2^x + \sqrt[3]{y}\right)$

(b) $f(x, y) = \cosh(x^2y) + y/\exp(x + y^2)$

(c) $f(x, y) = x\sec^2(x) + \tan(y^3)$

Problem 36 Let $f(x, y) = \dfrac{x}{x + y^2}$. Calculate the following:

(a) f_x

(b) f_y

(c) ∇f

(d) the rate of change in f at $(1, 3)$ in the $\langle 3, 4 \rangle$ direction

Problem 37 Let $f(x, y) = 3 + xy + x^3$. Consider the point $p = (2, 1)$. Find unit-vector(s) based at p which point in the direction in which f changes at the rate of 12 if possible. Also, if possible, find the unit-vectors which point in the direction in which f changes at rate 15 at p.

Problem 38 Let $f(x,y) = x^2y^3 + x + y$. Find the directions for the min/max rates of change of f at $(1,2)$. If L is a line through $(1,2)$ which is perpendicular to the directions for which the maximum and minimum rates of change of f at $(1,2)$ occur then what special property do we observe for $f(x,y)$ where $(x,y) \in L$ and (x,y) is very close to $(1,2)$?

Problem 39 Let $w = x^2 e^{xyz} + y^z$. Calculate $\partial_x w$, $\partial_y w$ and $\partial_z w$. Also, find ∇w.

Problem 40 Let $f(x,y) = x - y$. Calculate ∇f. Plot level curves for $f(x,y) = k$ where $k = -3, -2, -1, 0, 1, 2, 3$. Also plot ∇f. Explain the relation between the level curves and the gradient vector field.

Problem 41 The total differential of a function $f(x,y)$ is simply $df = (\partial_x f)dx + (\partial_y f)dy$. Calculate:

(a) df for $f(x,y) = \sqrt{x^2 + y^2}$

(b) dg for $g(x,y) = x^2 + y^2$

(c) relate f and g and df and dg. Explain the pattern.

Problem 42 Consider x, y the cartesian coordinate functions on $\mathbb{R}^2$. Prove from the limit-based definition of partial derivatives that $\frac{\partial y}{\partial x} = 0$ and $\frac{\partial x}{\partial x} = 1$.

Problem 43 Show that $u = e^x \cosh(y)$ and $v = e^x \sinh(y)$ are solutions of $u_{xx} - u_{yy} = 0$.

Problem 44 Suppose the temperature T is a function of the coordinates x, y in a large plane of battle. Furthermore, suppose the enemy ninja is carefully building a large attack by molding chakra over some time. During the preparation of the attack the enemy is vulnerable to your attack. Knowing this he has obscured your field of vision with multiple smoke bombs. However, the mass of energy building actually heats the ground. Fortunately one of your ninja skills is temperature sensitivity. You extrapolate from the temperature of the ground near your location that the temperature function has the form $T(x,y) = 50 - x + 6y$. In what direction should you attack?

Problem 45 Suppose

$$\vec{F}(x,y,z) = \langle yz + \sin(x), xz + e^y, 3 + xy \rangle.$$

What scalar function f yields $\vec{F}$ as a gradient vector field? Find f such that $\nabla f = \vec{F}$. *(here we have to work backwards, write down what you want and guess, by the way, the function $-f$ is the pontial energy function for the force field $\vec{F}$.)*

Bonus Problem: [use of technology to solve algebraic and/or transcendental equation that the problem suggests] The temperature in an air conditioned room is set at 65. A ninja with expert ocular jutsu disguises himself in plain sight by bending light near him with his art. However, his art does not extend to the infared spectrum and his body heat leaves a signature variation in the otherwise constant room temperature. In particular,

$$T(x,y,z) = 33 \exp\left[-(x-3)^2 - (y-4)^2 - (z-1)^2\right] + 65.$$

Shino searches for the cloaked ninja by sending insect scouts which are capable of sensing a change in temperature as minute as 0.1 degree per meter. How close do the scout insects have to get before they sense the hidden ninja? (also, where is the hidden ninja and what is his body temperature on the basis of the given T which is in meters and degrees Farenheight)