

Copying answers and steps is strictly forbidden. Evidence of copying results in zero for copied and copier. Working together is encouraged, share ideas not calculations. Explain your steps. This sheet must be printed and attached to your assignment as a cover sheet. The calculations and answers should be written neatly on one-side of paper which is attached and neatly stapled in the upper left corner. Box your answers where appropriate. Please do not fold. Thanks!

**Problem 61** Your signature below indicates you have:

- (a.) I have read §4.4 – 4.7 of Cook: \_\_\_\_\_.
- (b.) I have attempted homeworks from Salas and Hille as listed below: \_\_\_\_\_.

The following homeworks from the text are good rudimentary skill problems. These are not collected or graded. However, they are all usually odd problems thus there are answers given within Salas, Hille and Etgen's text:

§ 15.3 #'s 1, 5, 9, 15, 21, 27, 31, 33, 35, 36, 37, 39, 43, 45, 51, 57

§ 15.4 #'s 1, 5, 11, 13, 17, 19, 25, 27, 35

§ 15.7 #'s 1, 5, 11, 33

§ 15.8 #'s 1, 5, 15, 17, 29

§ 15.9 #'s 1, 5, 7, 9, 27

**Problem 62** Suppose Paccun speeds towards the base of a valley with paraboloid shape given by the equation  $z = 5x^2 + 3y^2$ . What is the direction of steepest **descent** at the point  $(1, 1, 8)$ ?

**Problem 63** Find the best affine<sup>1</sup> approximation of each object at the given point. Also, write either an equation or a parametrization of the tangent space in each case ("space" could mean line, surface, space curve or other things...)

- (a.)  $f(x) = x^2$  at  $a = 2$
- (b.)  $f(x, y) = x^2 - 2xy$  at  $(3, 4)$
- (c.)  $\vec{r}(t) = \langle t, 3, t^2 2^t \rangle$  at  $t = 0$

**Problem 64** Suppose  $z = x^2 + y^2$  and  $x = \exp[g(t)]$  and  $y = h(t^2 + 1)$  for some differentiable functions  $g, h$ . Calculate  $dz/dt$  by the chain rule(s).

**Problem 65** Let  $z = \sin(x^2 + y^2)$  and  $x = 2st$  and  $y = s^2 - t^2$ . Calculate  $\frac{\partial z}{\partial s}$  and  $\frac{\partial z}{\partial t}$ .

**Problem 66** Let  $x = u^2 - v^2$  and  $y = 7u - v$  and  $z = 3 \sinh(uv)$  and  $w = ze^{-xy}$  calculate  $w_u$  and  $w_v$ .

**Problem 67** Consider  $\vec{A}(x, y) = \langle x^2 + y^2, 2xy, \sin(x^3 + y) \rangle$  with  $x = \alpha^2 + \beta$  and  $y = \sqrt{\alpha^2 + \beta^2}$ . Calculate  $\partial_\alpha \vec{A}$  and  $\partial_\beta \vec{A}$ .

<sup>1</sup>this just means a linear function plus some constant

- Problem 68** Suppose a car speeds over a hill with equation  $x^2 + 4xy + z^2 = 4$ . If at the point with  $x = 1m$  and  $y = 0.5m$  the car has an  $x$ -velocity of  $10m/s$  and a  $y$ -velocity of  $20m/s$  then what  $z$ -velocity does the car have? (assume the car stays on hill and  $z > 0$ )
- Problem 69** Suppose  $S$  is the level surface defined by  $F(x, y, z) = (x^2 + y^2)^2 - z^2$ . Show that  $\gamma(t) = \langle \sin t \sqrt{\cosh t}, \cos t \sqrt{\cosh t}, \sinh t \rangle$  is a curve on  $S$ . Furthermore, show that  $\gamma'(t)$  is orthogonal to  $\nabla F(\gamma(t))$ .
- Problem 70** Let  $S_1 : x^2 + y^2 + z^2 = 3$  and  $S_2 : x + y + z = 1$ . If  $C$  is formed by the intersection of  $S_1$  and  $S_2$  then parametrize  $C$  and relate the direction of the normal to  $S_1$  and the normal of  $S_2$  to the direction of  $C$  at  $(1, 1, -1)$ .
- Problem 71** Suppose  $S$  is the surface defined by  $F(x, y, z) = xyz = 6$ . Find the equation of the tangent plane and the parametrization of the normal line through  $(1, 2, 3)$ .
- Problem 72** Find a parametrization  $\vec{X}$  of  $S$  from the previous problem which provides a patch in the locality of  $(1, 2, 3)$ . Use  $\alpha, \beta$  for your parameters and find the normal-vector field  $\vec{N}(\alpha, \beta)$  by computing  $\vec{N}(\alpha, \beta) = \frac{\partial \vec{X}}{\partial \alpha} \times \frac{\partial \vec{X}}{\partial \beta}$ . Do you obtain the same normal vector at  $(1, 2, 3)$  with this patch?
- Problem 73** Label the solution set of  $y^2 = x - z^2$  as  $M$ .
- present  $M$  as a level-surface for some function  $F$ . Explicitly state the formula for  $F$ . Find the normal vector field on  $M$ .
  - parametrize  $M$  and once more find the normal vector field. This time find  $\vec{N}$  explicitly in terms of your chosen parameters.
- Problem 74** Let  $w = x^2 + y^3 + z^4$ . Calculate  $\frac{\partial w}{\partial x}$  given that  $y^2 = x^3 + e^z$  in the following cases:
- using  $x, y$  as independent variables; in precise notation, calculate  $\left(\frac{\partial w}{\partial x}\right)_y$ ,
  - using  $x, z$  as independent variables; in precise notation, calculate  $\left(\frac{\partial w}{\partial x}\right)_z$
- Problem 75** You are given  $dz + dw = x^2 dx + xy dy$  and  $dz - dw = e^x dx + \sin(x + w) dy$ . Calculate  $\left(\frac{\partial z}{\partial x}\right)_y$ . You may leave the answer in terms of  $x, y$  and  $w$  despite the fact that  $w$  is viewed as a dependent variable in this calculation.
- Problem 76** Suppose you know  $x = -2 \pm 0.04$  and  $y = 2 \pm 0.03$  what are the (approximate) corresponding polar coordinate ranges.
- Problem 77** Show that if  $f$  is differentiable at each point of the line segment  $\overline{PQ}$  and  $f(P) = f(Q)$  then there exists a point  $R$  between  $P$  and  $Q$  for which  $\nabla f(R)$  is orthogonal to  $\overrightarrow{PQ}$ .
- Problem 78** If  $\vec{A}(x, y) = \langle 1 + e^y, xe^y + y^2 \rangle$  then decide if there exists  $f$  such that  $\nabla f = \vec{A}$ . If there does indeed exist such  $f$  then calculate its formula.
- Problem 79** Solve the exact differential equation  $\left(\frac{y}{x} + 6x\right) dx + (\ln x - 2) dy = 0$ .
- Problem 80** Solve the exact differential equation  $\left(e^x + \ln y + \frac{y}{x}\right) dx + \left(\frac{x}{y} + \ln x + \sin y\right) dy = 0$ .

**Bonus:** The method of characteristics is one of the many calculational techniques suggested by the total differential. The idea is simply this: given  $dx/dt = f(x, y)$  and  $dy/dt = g(x, y)$  we can solve both of these for  $dt$  to eliminate time. This leaves a differential equation in just the cartesian coordinates  $x, y$  and we can usually use a separation of variables argument to solve for the level curves which the solutions to  $dx/dt = f(x, y)$  and  $dy/dt = g(x, y)$  parametrize. **Use the technique just described to solve**

$$\frac{dx}{dt} = y \quad \& \quad \frac{dy}{dt} = -2x.$$

Continuing, suppose that the force  $\vec{F} = q(\vec{v} \times \vec{B} + \vec{E})$  is the net-force on a mass  $m$ . Furthermore, suppose  $\vec{B} = B\hat{z}$  and  $\vec{E} = E\hat{z}$  where  $E$  and  $B$  are constants. **Find the equations of motion** in terms of the initial position  $\vec{r}_o = \langle x_o, y_o, z_o \rangle$  and velocity  $\vec{v}_o = \langle v_{ox}, v_{oy}, v_{oz} \rangle$  by **solving the differential equations** given by  $\vec{F} = m\frac{d\vec{v}}{dt}$ . If  $E = 0$  and  $v_{oz} = 0$  then find the radius of the circle in which the charge  $q$  orbits.

*Hint: first solve for the velocity components via the technique from the initial part of this Bonus problem then integrate to get the components of the position vector.*