Same instructions as in Mission 1. See Mission 1 for details.

Problem 46 Your PRINTED NAME below indicates you have:

(a.) Your PRINTED NAME below indicates you have:

(a.) I have read Chapter 4 of Cook: ____________________.

(b.) I have attempted 10 problems either from Stewart(see below) or the end of chapter problems in my notes:________________.

§ 14.4 #'s 1, 3, 5, 11, 13, 15, 25, 27, 42

§ 14.5 #'s 1, 3, 5, 7, 11, 15, 17, 21, 25, 45*, 49*, 53*

§ 14.6 #'s 5, 7, 9, 15, 17, 21, 23, 25, 33, 37, 41, 43, 45, 51, 67

Problem 47 Suppose Paccun speeds towards the base of a valley with shape given by the equation $z = 5x^2 + xy + 3y^2$ where x, y, z are in meters.

(a) What is the direction of steepest **descent** at the point $(1, -1, 7)$?

(b) Assuming Paccun stays on the surface of the valley, if $dx/dt = 3\,m/s$ and $dy/dt = 4m/s$ then what is Paccun's speed[1] ?

Problem 48 Label the solution set of $y^2 = x - z^2$ as M.

(a.) present M as a level-surface for some function F. Explicitly state the formula for F. Find the normal vector field on M.

(b.) Parametrize M for $x > 0$ by using $x = t^2$ and $y = t\cos\beta$. Find the parametrization of z and then calculate $\vec{N}$ explicitly in terms of the given parameters.

Problem 49 Suppose $z = e^{x^2} + \sin(xy)$ and $x = \exp[g(t)]$ and $y = h(t^2 + 1)$ for some differentiable functions g, h. Calculate dz/dt by the chain rule(s). Your answer will have the unknown functions h and g.

Problem 50 Let $w = z^2 + \exp(x^3 + y^4)$ and $x = 2st$, $y = s^2 - t^2$ and $z = s + t$. Calculate $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$. You may use the intermediate variables x, y and z in the formulation of your answer.

Problem 51 Let $S_1 : x^2 + y^2 + z^2 = 4$ and $S_2 : x^2 + y^2 = 2$. If C is formed by the intersection of S_1 and S_2 then parametrize C and relate the direction of the normal to S_1 and the normal of S_2 to the direction of C (at points of C naturally).

Problem 52 Let $w = x^2 + y^3 + z^4$. Calculate $\frac{\partial w}{\partial x}$ given that $y^2 = x^3 + e^z$ in the following cases:

(a) using x, y as independent variables; in precise notation, calculate $\left(\frac{\partial w}{\partial x}\right)_y$,

(b) using x, z as independent variables; in precise notation, calculate $\left(\frac{\partial w}{\partial x}\right)_z$

[1] recall, speed is defined as the magnitude of the velocity

Problem 53 You are given $dz + dw = x^2dx + xydy$ and $dz - dw = e^x dx + \sin(x+w)dy$. Calculate $\left(\frac{\partial z}{\partial x}\right)_y$. You may leave the answer in terms of x, y and w despite the fact that w is viewed as a dependent variable in this calculation.

Problem 54 The following problems are closely related:

(a) If $\vec{A}(x,y) = \langle 1+e^y,\ xe^y + y^2 \rangle$ then decide if there exists f such that $\nabla f = \vec{A}$. If there does indeed exists such f then calculate its formula.

(b) Solve $(1+e^y)\,dx + (\ln x - 2)\,dy = 0$.

Problem 55 Suppose $x^2 + 3xyz + z^3 = 10$. Show that $\frac{\partial z}{\partial y}\frac{\partial y}{\partial x}\frac{\partial x}{\partial z} = -1$. Notice, to understand this we must consider x, y, z both as dependent and independent to complete the calculation. Also, explain where the implicit function theorem allows us to view z locally as a graph of a function in x, y. What about y? Where can we view $y = y(x,z)$?

Problem 56 Fun with non-cartesian coordinate formulae: (these are only in my notes)

(a) Let $g(r,\theta) = r^2\theta^2$ calculate ∇g in terms of the polar coordinate frame; that is, express ∇g in terms of functions of r and θ as well as $\widehat{r}$ and $\widehat{\theta}$.

(b) Suppose $\vec{F} = \widehat{\rho}\rho^2 + \widehat{\phi}\frac{\cos\phi}{\rho} + \widehat{\theta}\frac{\theta e^\theta}{\rho\sin\phi}$. Find f such that $\nabla f = \vec{F}$.

Problem 57 Let $f : \mathbb{R}^n \to \mathbb{R}$ and $h : \mathbb{R} \to \mathbb{R}$ be smooth functions then the chain-rule combines first semester differentation and the gradient as follows: $\nabla h(f(\vec{r})) = h'(f(\vec{r}))\nabla f(\vec{r})$ where $\vec{r} = (x_1, x_2, \dots, x_n)$. Let $r = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$ and calculate the following with the help of the chain-rule just given:

(a) ∇r

(b) ∇r^2

(c) $\nabla(1/r)$

Problem 58 Let $u = 3x + 2y$ and $v = x + y$ define new variables for $\mathbb{R}^2$. Convert Laplace's equation $\phi_{xx} + \phi_{yy} = 0$ to the (u,v) coordinate system. It may be helpful to consider the chain rules in operator notation:

$$\frac{\partial}{\partial x} = \frac{\partial u}{\partial x}\frac{\partial}{\partial u} + \frac{\partial v}{\partial x}\frac{\partial}{\partial v} \qquad \& \qquad \frac{\partial}{\partial y} = \frac{\partial u}{\partial y}\frac{\partial}{\partial u} + \frac{\partial v}{\partial y}\frac{\partial}{\partial v}.$$

Problem 59 Show that if f is differentiable at each point of the line segment $\overline{PQ}$ and $f(P) = f(Q)$ then there exists a point R between P and Q for which $\nabla f(R)$ is orthogonal to $\overrightarrow{PQ}$.

Problem 60 Observe $F(x,y,z,t)$ has total differential $dF = (\partial_x F)dx + (\partial_y F)dy + (\partial_z F)dz + (\partial_t F)dt$. We say $Pdx + Qdy + Rdz + Sdt = 0$ is exact if there exists F for which $dF = Pdx + Qdy + Rdz + Sdt$. What 6 conditions must be met in order that $Pdx + Qdy + Rdz + Sdt = 0$ is an exact equation? Is $ydx + xdy + z^3dz + (t+x)dt = 0$ exact ?